**Exercise 6** Let $L = (V, C, R)$ with $V = \{w, y\}$, $C = \{d, e\}$ and $R = \{r, s\}$ where $r$ has arity 1 and $s$ has arity 2. Which of the following are atoms over $L$? Which are ground atoms? Justify your answers.

(a) $d(w, w)$  
(b) $r(d, e)$  
(c) $s(w, w)$  
(d) $r(y)$

**Exercise 7** Let $L = (V, C, R)$ with $V = \{x, y\}$, $C = \{\text{barack, michelle, craig, malia}\}$ and $R = \{\text{motherOf, parentOf, grandmotherOf}\}$, all with arity 2. Which of the Datalog facts (1) to (9) from Example 1.1.1 are atoms over $L$? Justify your answers.

**Exercise 8** Write a Datalog program which captures the following natural language sentences.

(a) If somebody is an orphan, then all his parents are dead.
(b) Every orphan is a human being.
(c) Somebody’s father is also that person’s parent.
(d) Harry Potter is an orphan.
(e) James Potter is the father of Harry Potter.

**Exercise 9** Give three distinct Herbrand interpretations for the following Datalog program, where $a, b$ are constants.

```
q(a)  
p(b)  
q(x) → p(x)  
q(y) ∧ p(y) → r(b)
```

**Exercise 10** Evaluate the following.

(a) $(p(x, y, x) ∧ q(x, y, y) ∧ r(y, y) → t(x))[x/a, y/b] = \ldots$
(b) $(p(x) ∧ q(x) → r(x))[x/c][x/d] = \ldots$
(c) $(q(a, x) ∧ p(x, y) ∧ q(y, a) → r(y))[x/a][x/b] = \ldots$
(d) $(p(x, x) ∧ q(x, y) → p(x, y))[y/b][y/c][x/b] = \ldots$

**Exercise 11** Which of the substitutions in Exercise 10 are ground substitutions?

**Exercise 12** Give the grounding of the Datalog program from Exercise 9.

**Exercise 13** Give a Herbrand model for the Datalog program in Exercise 9.

**Exercise 14** Give three distinct Herbrand models for the Datalog program $P$ consisting of the following rules.

```
p(a, b)  
q(c)  
p(x, y) → q(x)
```