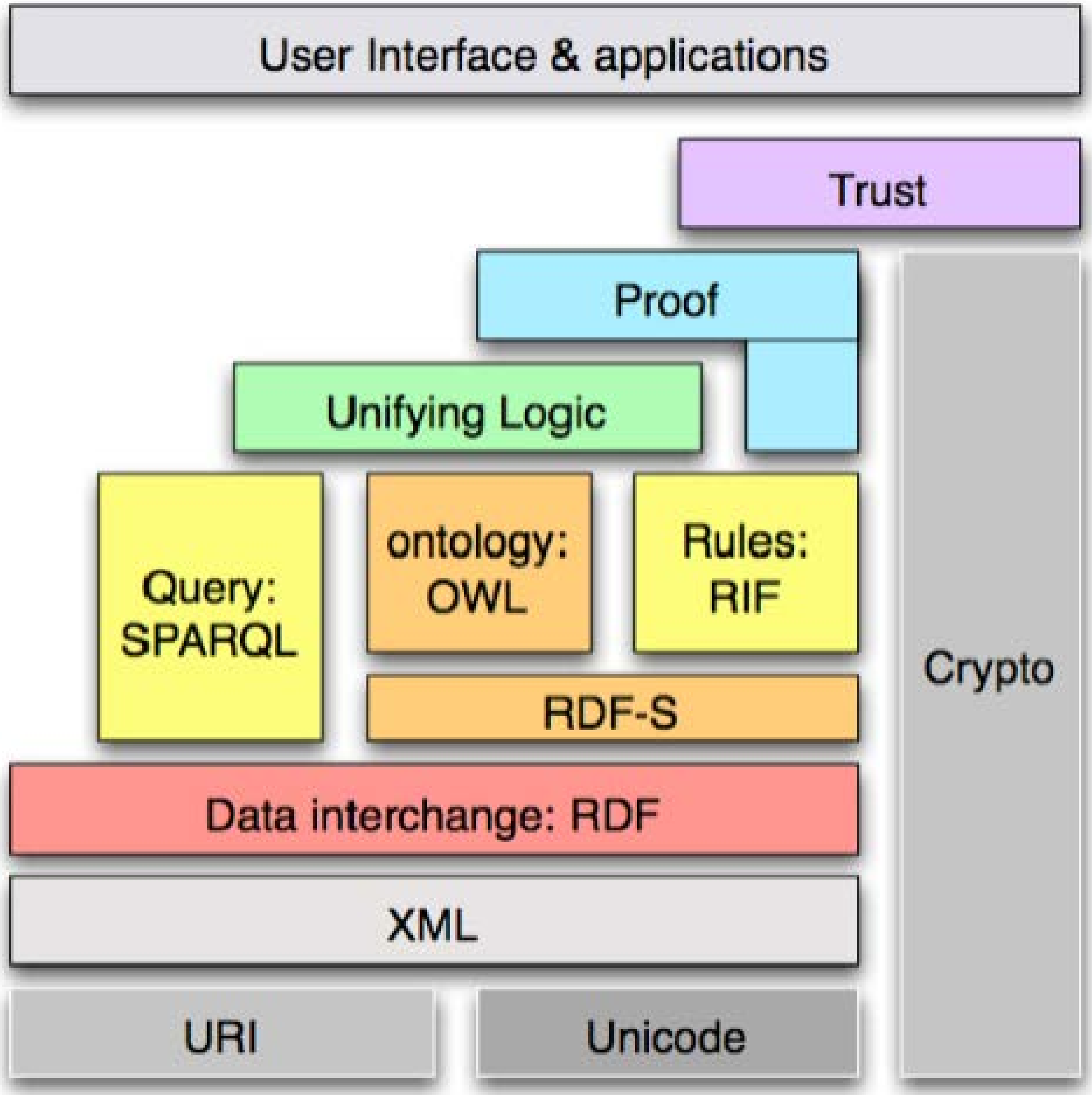
CS 2210: Logic for Computer Scientists
Description Logics

Pascal Hitzler
DaSe Lab for Data Semantics
Wright State University, Dayton, OH
http://www.pascal-hitzler.de

The Semantic Web Stack



OWL

DaSe Lab

Web Ontology Language (OWL)

- **W3C Recommendation since 2004**
- **OWL 2 since 2009**

- **based on description logics**
- **essentially, a decidable fragment of first-order predicate logic**

Description Logics (DLs)

classes/concepts	**unary predicates**
A, B, C	**A(x), B(X), C(x)**
roles/properties	**binary predicates**
R, S	**R(x,y), S(x,y)**
individuals	**constants**
a, b, c	**a, b, c**

Some DL constructors

class conjunction

C u D — C(x) ÆD(x)

existential restriction

9R.C — 9y (R(x,y) ÆC(y))

class inclusion/subsumption

C v D — C(x) → D(x)

C ´ D — C(x) $ D(x)

role chains

R_1± … ± R_n v R — R_1 (x,x_1) Æ… ÆR(x_n,x_{n+1}) → R(x,x_{n+1})

Some DL constructors

ThaiDish v 9contains.Nut

Nutallergic u 9eats.Nut v Unhappy

eats ± contains v eats

ThaiDish(x) → 9y (contains(x,y) ÆNut(y))

Nutallergic(x) Æ9y (eats(x,y) ÆNut(y)) → Unhappy(x)

eats(x,y) Æcontains(y,z) → eats(x,z)

Some DL constructors

ThaiDish v 9contains.Nut

Nutallergic u 9eats.Nut v Unhappy

eats ± contains v eats

inverse roles

R ´ S¯ **R(x,y) $ S(y,x)**

This logic is already undecidable! **(see e.g. [ISWC 2007])**

Name of the logic: ELRI

Decidability

Decidability is a central characteristics of description logics.

Retaining Decidability

1. **Disallow 9:**
   Essentially leads to OWL RL.
   Fragment of Datalog.
   Tractable (i.e., polynomial complexity).

2. **Disallow inverse roles:**
   Essentially leads to OWL EL.
   Akin "in spirit" to existential rules/Datalog+-.
   Tractable.

3. **Restrict recursion in role chains (a.k.a. *regularity* restriction):**
   With further constructors, leads to OWL DL, a.k.a. SROIQ.
   Decidable, but not tractable.

Further essential DL constructors

The following can be used in OWL EL (logic remains tractable).

Self

C v 9R.Self **C(x) → R(x,x)**

Can be used e.g. for typecasting.

nominals

{a} v C	**C(a)**	**a is a constant**
C v {a}	**C(x) → x=a**	
{a} ´ {b}	**→ a=b**	

$$A \sqcap \exists R.\{b\} \sqsubseteq C \text{ becomes } A(x) \wedge R(x,b) \rightarrow C(x)$$

Further essential DL constructors

The following are used in expressive (intractable) DLs

class negation

: C	**: C(x)**

class disjunction

C t D	**C(x) Ç D(x)**

universal restriction

8R.C	**8y (R(x,y) → C(y))**

There are some more of course.

Rules in OWL

$$A \sqsubseteq B \text{ becomes } A(x) \to B(x)$$

$$R \sqsubseteq S \text{ becomes } R(x,y) \to S(x,y)$$

$$A \sqcap \exists R.\exists S.B \sqsubseteq C \text{ becomes } A(x) \land R(x,y) \land S(y,z) \land B(z) \to C(x)$$

$$R \circ S \sqsubseteq T \text{ becomes } R(x,y) \land S(y,z) \to T(x,z)$$

$$A \sqcap \exists R.\{b\} \sqsubseteq C \text{ becomes } A(x) \land R(x,b) \to C(x)$$

Rolification

$$\text{Elephant}(x) \wedge \text{Mouse}(y) \rightarrow \text{biggerThan}(x, y)$$

- **Rolification of a concept A:** **A ´ 9R_A.Self**

$$\text{Elephant} \equiv \exists R_{\text{Elephant}}.\text{Self}$$

$$\text{Mouse} \equiv \exists R_{\text{Mouse}}.\text{Self}$$

$$R_{\text{Elephant}} \circ U \circ R_{\text{Mouse}} \sqsubseteq \text{biggerThan}$$

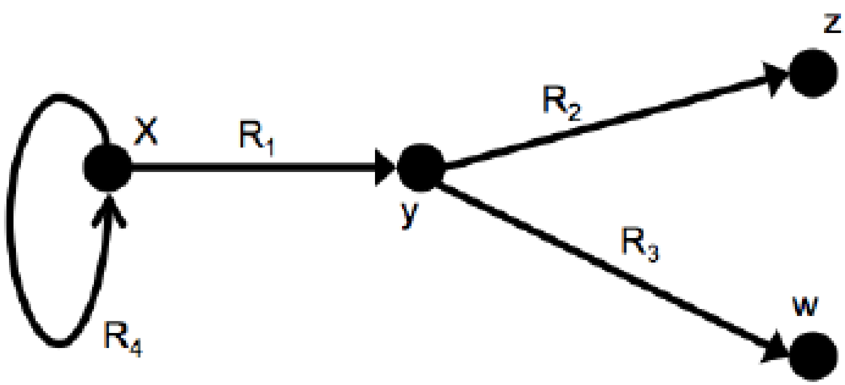
Rolification

$$\text{worksAt}(x,y) \wedge \text{University}(y) \wedge \text{supervises}(x,z) \wedge \text{PhDStudent}(z) \rightarrow \text{professorOf}(x,z)$$

$$R_{\exists\text{worksAt.University}} \circ \text{supervises} \circ R_{\text{PhDStudent}} \sqsubseteq \text{professorOf}.$$

Tree-shaped rules

$$R_1(x, y) \wedge C_1(y) \wedge R_2(y, w) \wedge R_3(y, z) \wedge C_2(z) \wedge R_4(x, x) \to C_3(x)$$

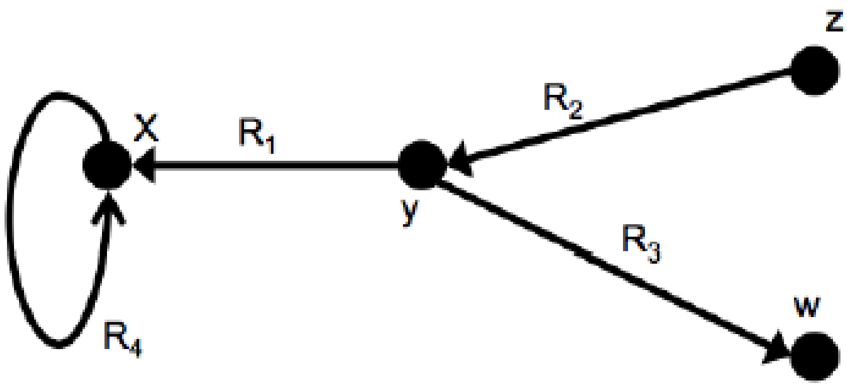


$$\exists R_1.(C_1 \sqcap \exists R_2.\top \sqcap \exists R_3.C_2) \sqcap \exists R_4.Self \sqsubseteq C_3$$

Acyclic Rules

DaSe Lab

$$R_1(y,x) \wedge C_1(y) \wedge R_2(w,y) \wedge R_3(y,z) \wedge C_2(z) \wedge R_4(x,x) \rightarrow C_3(x)$$

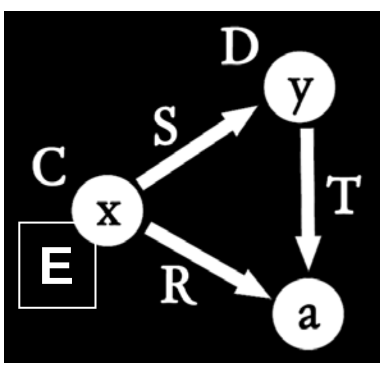


$$\exists R_1^-.(C_1 \sqcap \exists R_2^-.\top \sqcap \exists R_3.C_2) \sqcap \exists R_4.Self \sqsubseteq C_3$$

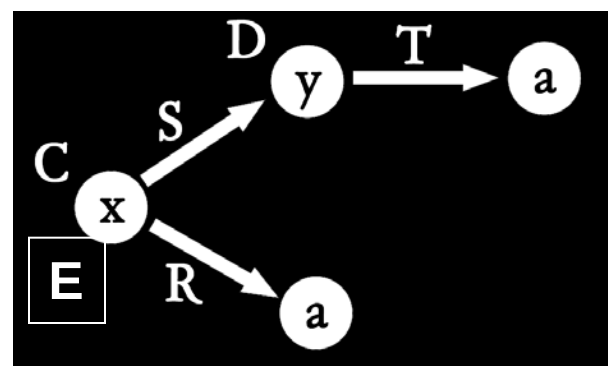
So how can we pinpoint this?

- Tree-shaped bodies
- First argument of the conclusion is the root

- C(x) ÆR(x,a) ÆS(x,y) ÆD(y) ÆT(y,a) ! E(x)
  - C u 9R.{a} u 9S.(D u 9T.{a}) v E



duplicating nominals is ok



Rule bodies as graphs

$$C(x) \wedge R(x,a) \wedge S(x,y) \wedge D(y) \wedge T(y,a) \rightarrow P(x,y)$$

$$a_1 \longleftarrow x \longrightarrow y \longrightarrow a_2$$

C u 9R.{a} v 9R1.Self

D u 9T.{a} v 9R2.Self

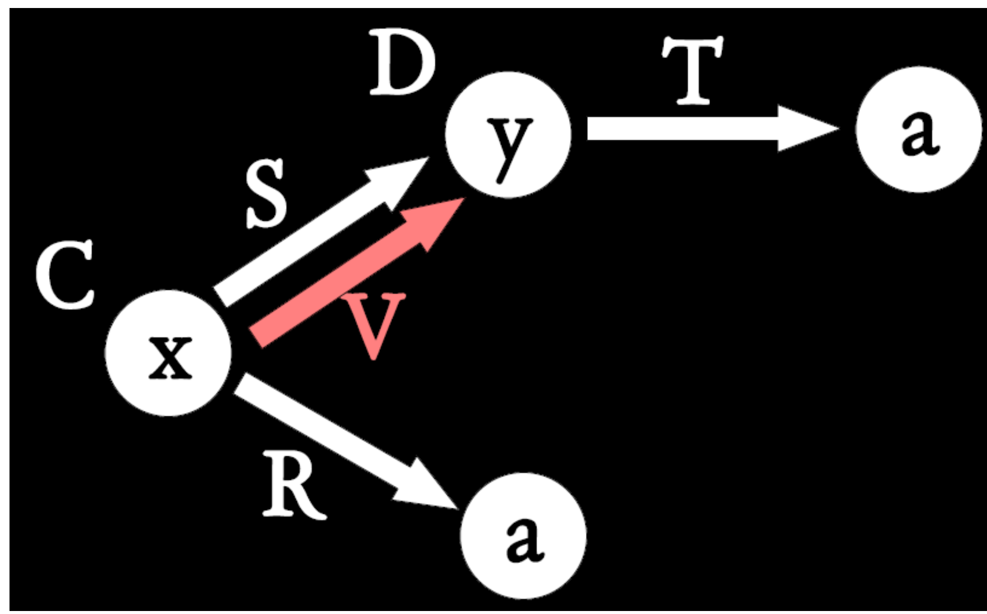
R1 ± S ± R2 v P

So how can we pinpoint this?

- **Tree-shaped bodies**
- **First argument of the conclusion is the root**

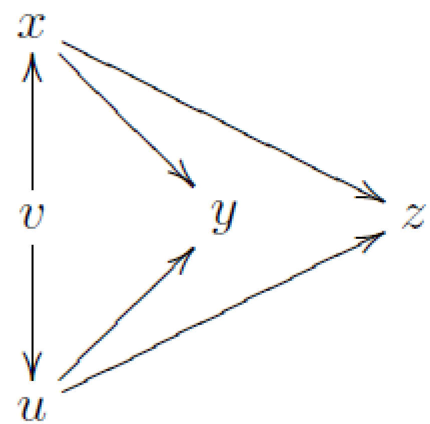
- **C(x) ÆR(x,a) ÆS(x,y) ÆD(y) ÆT(y,a) ! V(x,y)**

C u 9R.{a} v 9R1.Self
D u 9T.{a} v 9R2.Self
R1 o S o R2 v V



Rule bodies as graphs

$$\text{hasReviewAssignment}(v,x) \wedge \text{hasAuthor}(x,y) \wedge \text{atVenue}(x,z)$$
$$\wedge\ \text{hasSubmittedPaper}(v,u) \wedge \text{hasAuthor}(u,y) \wedge \text{atVenue}(u,z)$$
$$\rightarrow \text{hasConflictingAssignedPaper}(v,x)$$



with y,z constants (otherwise not expressible):

$$R_{\exists\text{hasSubmittedPaper}.(\exists\text{hasAuthor}.\{y\}\sqcap\exists\text{atVenue}.\{z\})} \circ \text{hasReviewAssignment}$$
$$\circ\ R_{\exists\text{hasAuthor}.\{y\}\sqcap\exists\text{atVenue}.\{z\}}$$
$$\sqsubseteq \text{hasConflictingAssignedPaper}$$

Conclusions

Description Logics

- Adopted as Semantic Web standard OWL
- Essentially, a fragement of first-order predicate logic, but decidable.

- Acyclic rules can be encoded.
- Existential quantifiers can be used, but in a restricted fashion.

- Conditions play together such that decidability is retained.

References

A tutorial:

- Adila A. Krisnadhi, Frederick Maier, Pascal Hitzler, OWL and Rules. In: A. Polleres, C. d'Amato, M. Arenas, S. Handschuh, P. Kroner, S. Ossowski, P.F. Patel-Schneider (eds.), Reasoning Web. Semantic Technologies for the Web of Data. 7th International Summer School 2011, Galway, Ireland, August 23-27, 2011, Tutorial Lectures. Lecture Notes in Computer Science Vol. 6848, Springer, Heidelberg, 2011, pp. 382-415.

Background reading:

- Pascal Hitzler, Markus Krötzsch, Sebastian Rudolph, Foundations of Semantic Web Technologies. Textbooks in Computing, Chapman and Hall/CRC Press, 2009. http://www.semantic-web-book.org/
- Pascal Hitzler, Markus Krötzsch, Bijan Parsia, Peter F. Patel-Schneider, Sebastian Rudolph, OWL 2 Web Ontology Language: Primer (Second Edition). W3C Recommendation, 11 December 2012. http://www.w3.org/TR/owl2-primer/

References

- Markus Krötzsch, Frederick Maier, Adila Alfa Krisnadhi, Pascal Hitzler, A Better Uncle For OWL – Nominal Schemas for Integrating Rules and Ontologies. In: S. Sadagopan, Krithi Ramamritham, Arun Kumar, M.P. Ravindra, Elisa Bertino, Ravi Kumar (eds.), WWW '11 20th International World Wide Web Conference, Hyderabad, India, March/April 2011. ACM, New York, 2011, pp. 645-654.
- Markus Krötzsch, Sebastian Rudolph, Pascal Hitzler, Description Logic Rules. In: Malik Ghallab, Constantine D. Spyropoulos, Nikos Fakotakis, Nikos Avouris (eds.), Proceedings of the 18th European Conference on Artificial Intelligence, ECAI2008, Patras, Greece, July 2008. IOS Press, 2008, pp. 80-84.
- Markus Krötzsch. Description Logic Rules. Studies on the Semantic Web, Vol. 008, IOS Press, 2010. http://www.semantic-web-studies.net/