A Proof that P≠NP

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Abstract
We demonstrate the separation of the complexity class NP from its subclass P.

Preliminaries

Preliminary definitions and background can be found in [Sudkamp, 2006], and the following are taken from [Sudkamp, 2006].

[Sudkamp, 2006, Section 8.7]: Every nondeterministic Turing Machine can be simulated by a deterministic Turing Machine. Hence, they give rise to the same notion of computability.

[Sudkamp, 2006, Definition 8.8.1]: A deterministic (k-tape) Turing Machine *enumerates* a language L if all of the following hold.

- The computation begins with all tapes blank.
- With each transition, the tape head on tape 1 (the output tape) remains stationary or moves to the right.
- At any point in the computation, the nonblank portion of tape 1 has the form
  B#u1#u2#...#uk# or B#u1#u2…#uk#v
  where u1,u2,… are in L and v is a string over the tape alphabet.
- A string u will be written on tape 1 preceded and followed by # if, and only if, u is in L.

[Sudkamp, 2006, Theorem 8.8.6]: A language is recursively enumerable if, and only if, it can be enumerated by a deterministic Turing Machine.

The following is easily shown from the above. We include a proof for completeness.

Theorem 1
A language is recursively enumerable if, and only if, it can be enumerated by a nondeterministic Turing Machine.

Proof.
By the results cited above, a language is recursively enumerable if, and only if, it can be enumerated by a deterministic Turing Machine, while deterministic Turing Machines can simulate nondeterministic ones (and vice versa). qed.

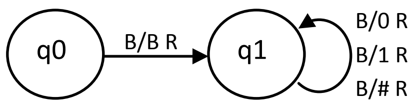
Results

We now proceed to the new results.

Theorem 2
Every set of non-negative integers is recursively enumerable.

Proof.
Let S be an arbitrary set of non-negative integers. Let L be the language containing exactly those strings over {0,1} which are binary representations of a number in S.

Now consider the following (1-tape) nondeterministic Turing Machine M, where q0 is the start state, and B stands for a blank read from the tape.



Obviously, there is a computation of M which produces L (and therefore S). By Theorem 1 we have that L, and therefore S, is recursively enumerable. Since S was chosen arbitrarily, any set of non-negative numbers is recursively enumerable.   qed.

Corollary 1
The set of all subsets of the non-negative integers is countable.

Proof.
Since every Turing Machine can be described by a finite string (or, use Gödel numbering), the set of all Turing Machines is countable. Since every subset of the non-negative integers can be enumerated by a Turing Machine (Theorem 2), the set of all these subsets must be countable.   qed.

Corollary 2
The theoretical foundations of Computer Science are contradictory.

Proof.
Georg Cantor has shown (using a diagonalization argument) that the set of all subsets of the non-negative integers is uncountable, which contradicts Corollary 1.   qed.

Corollary 3
P ≠ NP.

Proof.
Since the theoretical foundations of Computer Science are contradictory, the statement follows immediately.   qed.

References

[Sudkamp, 2006] Sudkamp, T.A. (2006). Languages and Machines. Addison Wesley, 3rd edition.