Local closed world semantics: grounded circumscription for description logics

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Abstract. We present an improved local closed world extension for description logics. It is based on circumscription, and deviates from previous circumscriptive description logics [1, 3] in that extensions of minimized predicates may contain only extensions of named individuals in the knowledge base. Besides an (arguably) higher intuitive appeal, the improved semantics is applicable to expressive description logics without loss of decidability.

Keywords: description logic, local closed world, circumscription

1 Introduction

The semantics of the Web Ontology Language OWL [4] (which is based on the description logic $\mathcal{SROIQ}$ [5]) adheres to the Open World Assumption (OWA), which means that statements which are *not* logical consequences of a given knowledge base are not necessarily considered false. The OWA is a very reasonable assumption to make in the World Wide Web context (and thus for Semantic Web applications), however situations naturally arise where it would be preferable to use the Closed World Assumption (CWA), that statements which are *not* logical consequences of a given knowledge base are always considered false. The CWA is applicable, e.g., when data is being retrieved from a database, or if data can otherwise be considered *complete* with respect to the application at hand (see, e.g., [2]).

As a consequence, efforts have been made to combine OWA and CWA for the Semantic Web, and knowledge representation languages which have both OWA and CWA modeling features are said to adhere to the *Local Closed World Assumption* (LCWA). Most of these combinations are derived from non-monotonic logics which have been studied in logic programming or on first-order predicate logic, and many of them have a *hybrid* character, meaning that they achieve the LCWA by combining, e.g. description logics with (logic programming) rules.

Of the approaches which provide a seamless (non-hybrid) integration of OWA and CWA, there are not that many, and each of them has its drawbacks. This is despite the fact that the modeling task, from the perspective of the application developer, seems rather simple: Users would want to specify, simply, that individuals in the extension of a predicate should be exactly those which are *necessarily required* to be in it, i.e., extensions should be *minimized*. Thus, what is needed for applications is a simple, intuitive approach to closed world modeling.

Among the primary approaches to non-monotonic reasoning, there is exactly one approach which employs the minimization idea in a very straightforward and intuitively simple manner, namely *circumscription* [7]. However, a naive transfer of the circumscription approach to description logics leads to undecidability for expressive description logics if role minimization is allowed [1, 3]. Our idea to remedy this is simple yet effective: we modify the circumscription approach from [1, 3] by adding the additional requirement that extensions of minimized predicates may only contain named individuals (or pairs of such, for roles).

The paper is structured as follows. In Section 2 we introduce the semantics of grounded circumscription. In Section 3 we show that the resulting language is decidable. In Section 4 we conclude with a discussion of further work.

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2 Grounded Circumscription

We now describe a very simple way for ontology designers to model local closed world aspects in their ontologies: simply use a description logic (DL) knowledge base (KB) as usual, and augment it with *meta*-information which states that some predicates (concept names or role names) are *closed*. Semantically, those predicates are considered minimized, i.e. their extensions contain only what is absolutely required, and furthermore only contain *known* (or *named*) individuals, i.e., individuals which are explicitly mentioned in the KB. In the case of concept names, the idea of restricting their extensions only to known individuals is similar to the notion of nominal schema [6], while the minimization idea is borrowed from circumscription [7], one of the primary approaches to non-monotonic reasoning.

Indeed, the ideas of carrying over circumscription to DLs is not new, and has already been described in [1, 3]. In particular, they divided the predicates (concept names and role names) in the KB into three disjoint sets of *minimized*, *fixed* and *varying* predicates. These sets together with some preference relation on interpretations made up a *circumscription pattern*. The preference relation allows us to pick the *minimal* models as the *preferred* models with respect to inclusion of the extension of the minimized predicates.

Our formalism simplifies the circumscription approach by restricting our attention to models in which the extensions of the minimized predicates may only contain known individuals from the KB. Moreover, we divide predicates in the KB only into two disjoint sets of minimized and non-minimized predicates.[1]

[1] Fixed predicates can be simulated in the original circumscriptive DL approach if negation is available, i.e., for fixed class names, class negation is required, while for fixed role names, role negation is required. The latter can be added to expressive DLs without jeopardizing decidability [6, 8].

The non-minimized predicates would be viewed as varying in the more general circumscription formalism mentioned above.

Let N_C, N_r, and N_I be three disjoint, countably infinite sets of *concept names*, *role names*, and *individual names*, resp. Let $\mathcal{L}$ be a standard description logic based on the signature formed from N_C, N_R, and N_I. In addition, we define an $\mathcal{L}$*-KB* as a set of concept inclusion axioms $C \sqsubseteq D$ where $C, D \in \mathsf{N}_C$, and assertions of the form $C(a)$ and $r(a, b)$ where $C \in \mathsf{N}_C, r \in \mathsf{N}_r$ and $a, b \in \mathsf{N}_I$.

The semantics for $\mathcal{L}$ is defined in terms of *interpretations* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where $\Delta^{\mathcal{I}}$ is a non-empty set called the *domain* and $\cdot^{\mathcal{I}}$ is an *interpretation function* that maps each concept name to a subset of $\Delta^{\mathcal{I}}$, each role name to a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and each individual name to an element of $\Delta^{\mathcal{I}}$. An interpretation $\mathcal{I}$ is extended to complex concepts and roles in the usual way for $\mathcal{L}$. We say that $\mathcal{I}$ *satisfies* (*is a model of*): an axiom $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$; an axiom $C(a)$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$; and an axiom $r(a, b)$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$. We also say that $\mathcal{I}$ *satisfies* (*is a model of*) an $\mathcal{L}$-KB K if it satisfies every axioms in K. A concept C is *satisfiable with respect to* an $\mathcal{L}$-KB K if there is a model $\mathcal{I}$ of K such that $C^{\mathcal{I}} \neq \emptyset$.

The non-monotonic feature of the formalism is given by restricting models of an $\mathcal{L}$-KB such that the extensions of closed predicates may only contain individuals (or pairs of them) which are explicitly occurring in the KB, plus a minimization of the extensions of these predicates. We define a function Ind that maps each $\mathcal{L}$-KB to the set of individual names it contains, i.e., given an $\mathcal{L}$-KB K, $\mathsf{Ind}(K) = \{b \in \mathsf{N}_I \mid b \text{ occurs in } K\}$. Among all possible models of K that are obtained by the aforementioned restriction to $\mathsf{Ind}(K)$, we then select a model that is minimal w.r.t. concept inclusion or role inclusion.

Definition 1. *A* GC-$\mathcal{L}$-knowledge base *(KB)—GC stands for* grounded circumscription*—is a pair* (K, M) *where* K *is an* $\mathcal{L}$*-KB and* $M \subseteq \{A \in \mathsf{N}_C \mid A \text{ occurs in } K\} \cup \{r \in \mathsf{N}_r \mid r \text{ occurs in } K\}$. *For every concept name and role name* $W \in M$, *we say that* W *is* closed with respect to K.

Definition 2. *Let* (K, M) *be a GC-*$\mathcal{L}$*-KB and* $\mathcal{I}$ *and* $\mathcal{J}$ *be two models of* K. *We say that* $\mathcal{I}$ *is* smaller than $\mathcal{J}$ w.r.t. M *iff all of the following hold:*
- $\Delta^{\mathcal{I}} = \Delta^{\mathcal{J}}$ *and* $a^{\mathcal{I}} = a^{\mathcal{J}}$ *for every* $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$*;*
- $W^{\mathcal{I}} \subseteq W^{\mathcal{J}}$ *for every* $W \in M$*; and*
- *there exists a* $W \in M$ *such that* $W^{\mathcal{I}} \subset W^{\mathcal{J}}$

We now define models of GC-$\mathcal{L}$-KB as follows.

Definition 3. *An interpretation* $\mathcal{I}$ *is a* GC-model *of a GC-*$\mathcal{L}$*-KB* (K, M) *if all of the following hold:*
- $\mathcal{I}$ *is a model of* $\mathcal{K}$*;*
- *for each concept name* $A \in M$, $A^{\mathcal{I}} \subseteq \{b^{\mathcal{I}} \mid b \in \mathsf{Ind}(\mathcal{K})\}$*;*
- *for each role name* $r \in M$, $r^{\mathcal{I}} \subseteq \{b^{\mathcal{I}} \mid b \in \mathsf{Ind}(\mathcal{K})\} \times \{b^{\mathcal{I}} \mid b \in \mathsf{Ind}(\mathcal{K})\}$*; and*
- $\mathcal{I}$ *is minimal w.r.t.* M, *i.e., there is no model* $\mathcal{J}$ *of* K *such that* $\mathcal{J}$ *is smaller than* $\mathcal{I}$ *w.r.t.* M.

The notion of logical consequence is defined as usual: An axiom α is a logical consequence (a *GC-inference*) of a given GC-$\mathcal{L}$-KB (K, M) if and only if α is true in all GC-models of (K, B). Note, that every GC-model of a KB is also a circumscriptive model, hence every circumscriptive inference is also a valid GC-inference.

3 Decidability Considerations

As noted earlier, circumscription in many expressive DLs is undecidable [1]. Undecidability even extends to the basic DL $\mathcal{ALC}$ when non-empty TBoxes are considered and roles are allowed as minimized predicates. Such a bleak outlook would greatly discourage useful application of circumscription, despite the fact that there is a clear need of such a formalism to model LCWA.

Our formalism aims to fill this gap by offering a simpler approach to circumscription in DLs that is decidable provided that the underlying DL is also decidable. The decidability result is obtained due to the imposed restriction of minimized predicates to known individuals in the KB as specified in Definition 3. Let $\mathcal{L}$ be any standard DL. We consider the following reasoning task of *GC-KB satisfiability*: "given a GC-$\mathcal{L}$-KB (K, M), does (K, M) have a GC-model?" and show in the following that this is decidable. Note that other basic reasoning tasks can usually be reduced to this task [1, 3].

Assume that $\mathcal{L}$ is any (standard) DL, e.g., $\mathcal{ALCOB}(\times)$, featuring nominals, concept disjunction, concept products and role disjunctions.[2] We show that GC-KB satisfiability in $\mathcal{L}$ is decidable if satisfiability in $\mathcal{L}$ is decidable.

Let (K, M) be a GC-$\mathcal{L}$-KB. We assume that $M = M_A \cup M_r$ where $M_A = \{A_1, \ldots, A_n\}$ is the set of minimized concept names and $M_r = \{r_1, \ldots, r_m\}$ is the set of minimized role names. Now define a family of $(n + m)$-tuples as

$$\mathcal{G}_{(K,M)} = \{(X_1, \ldots, X_n, Y_1, \ldots, Y_m) \mid X_i \subseteq \mathsf{Ind}(K), Y_j \subseteq \mathsf{Ind}(K) \times \mathsf{Ind}(K)\}$$

with $1 \leq i \leq n, 1 \leq j \leq m$. Note that there are

$$\left(2^{|\mathsf{Ind}(K)|}\right)^n \cdot \left(2^{\mathsf{Ind}(K)^2}\right)^m = 2^{n \cdot |\mathsf{Ind}(K)| + m \cdot |\mathsf{Ind}(K)|^2} \tag{1}$$

of such tuples; in particular note that $\mathcal{G}_{(K,M)}$ is a finite set.

Now, given (K, M) and some $G = (X_1, \ldots, X_n, Y_1, \ldots, Y_m) \in \mathcal{G}_{(K,M)}$, let K_G be the $\mathcal{L}$-KB consisting of all axioms in K together with all of the following axioms, where the A_i and r_j are all the predicates in M—note that we require role disjunction and concept products for this.

$$A_i \equiv \bigsqcup \{a\} \quad \text{for every } a \in X_i \text{ and } i = 1, \ldots, n$$
$$r_j \equiv \bigsqcup (\{a\} \times \{b\}) \quad \text{for every pair } (a, b) \in Y_j \text{ and } j = 1, \ldots, m$$

Then the following result clearly holds.

[2] For concept products, see [6]—they can be eliminated if role constructors are available. For role disjunctions, see [8], where it is shown, amongst other things, that $\mathcal{ALCQIOB}$ is decidable.

Lemma 1. *Let (K, M) be a GC-$\mathcal{L}$-KB. If (K, M) has a GC-model I, then there exists $G \in \mathcal{G}_{(K,M)}$ such that K_G has a (classical) model J which coincides with I on all minimized predicates. Likewise, if there exists $G \in \mathcal{G}_{(K,M)}$ such that K_G has a (classical) model J, then (K, M) has a GC-model I which coincides with J on all minimized predicates.*

Now consider the set

$$\mathcal{G}'_{(K,M)} = \{G \in \mathcal{G}_{(K,M)} \mid K_G \text{ has a (classical) model}\},$$

and note that this set is finite and computable in finite time since $\mathcal{G}_{(K,M)}$ is finite and $\mathcal{L}$ is decidable. Furthermore, consider $\mathcal{G}'_{(K,M)}$ to be ordered by the pointwise ordering $\prec$ induced by $\subseteq$. Note that the pointwise ordering of the finite set $\mathcal{G}'_{(K,M)}$ is also computable in finite time.

Theorem 1. *Let (K, M) be a GC-$\mathcal{L}$-KB, and let*

$$\mathcal{G}''_{(K,M)} = \{G \in \mathcal{G}'_{(K,M)} \mid G \text{ is minimal in } (\mathcal{G}'_{(K,M)}, \prec)\}.$$

Then (K, M) has a GC-model if and only if $\mathcal{G}''_{(K,M)}$ is non-empty.

Proof. This follows immediately from Lemma 1 together with the following observation: Whenever K has two GC models I and J such that I is smaller than J, then there exist $G_I, G_J \in \mathcal{G}'_{(K,M)}$ with $G_I \prec G_J$ such that K_{G_I} and K_{G_J} have (classical) models I' and J', respectively, which coincide with I, respectively, J, on the minimized predicates.

Corollary 1. *GC-KB-satisfiability is decidable.*

Proof. This follows from Theorem 1 since the set $\mathcal{G}''_{(K,M)}$, for any given GC-KB (K, M), can be computed in finite time, i.e., it can be decided in finite time whether $\mathcal{G}''_{(K,M)}$ is empty.

Some remarks on complexity are as follows. Assume that the problem of deciding KB satisfiability in $\mathcal{L}$ is in the complexity class C. Observe from equation (1) that there are exponentially many possible choices of the $(n+m)$-tuples in $\mathcal{G}_{(K,M)}$ (in the size of the input knowledge base). Computation of $\mathcal{G}'_{(K,M)}$ is thus in $\textsc{Exp}^{\mathrm{C}}$, and subsequent computation of $\mathcal{G}''_{(K,M)}$ is also in $\textsc{Exp}$. We thus obtain the following upper bound.

Proposition 1. *GC-KB satisfiability is in $\textsc{Exp}^{C}$, where C is the complexity class of the DL under consideration.*

Observe that the decidability proof gives rise to a straightforward implementation procedure, however this is certainly not a smart algorithm.

4 Conclusion and Outlook

We have provided a new approach for incorporating the LCWA into description logics. Our approach, grounded circumscription, is a variant of circumscriptive description logics which avoids two major issues of the original approach: Extensions of minimized predicates can only contain named individuals, and we retain decidability even for very expressive description logics while we can allow for the minimization of roles.

A primary theoretical task is to investigate the complexity of our modified approach, but it can be expected that it is not going to be worse than the previous circumscription proposal. In fact, lower complexities should result in some cases, which may yield to tractable or data-tractable fragments.

Likewise, it should be possible to adapt the tableaux algorithm for circumscriptive description logics from [3] to our setting, and there may even be more efficient procedures.

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