

\mathcal{EL} -ifying Ontologies ^{*}

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Abstract. The OWL 2 profiles are fragments of the ontology language OWL 2 for which standard reasoning tasks are feasible in polynomial time. Many OWL ontologies, however, contain a typically small number of out-of-profile axioms, which may have little or no influence on reasoning outcomes. We investigate techniques for rewriting axioms into the EL and RL profiles of OWL 2. We have tested our techniques on both classification and data reasoning tasks with encouraging results.

1 Introduction

Description Logics (DLs) are a family of knowledge representation formalisms underpinning the W3C standard ontology languages OWL and OWL 2. State-of-the-art DL reasoners such as Pellet [18], JFact, FaCT⁺⁺ [21], RacerPro [9], and HermiT [15] are highly-optimised for classification (i.e., the problem of computing all subsumption relationships between atomic concepts in an ontology) and have been exploited successfully in many applications. In a recent large-scale evaluation campaign, these reasoners exhibited excellent performance on a corpus of more than 1,000 ontologies, as they were able to classify 75%-85% of the corpus in less than 10 seconds when running on stock hardware [8,3].

However, notwithstanding extensive research into optimisation techniques, DL reasoning remains a challenge in practice. Indeed, the aforementioned evaluation also revealed that many ontologies are still hard for reasoners to classify. Furthermore, due to the high worst-case complexity of reasoning, systems are inherently not robust, and even minor changes to ontologies can have a significant effect on performance. Finally, the limitations of DL reasoners become even more apparent when reasoning with ontologies and large datasets.

These issues have motivated a growing interest in lightweight DLs: weaker logics that enjoy more favourable computational properties. OWL 2 specifies several profiles (language fragments) based on lightweight DLs [14]: OWL 2 EL (or just EL) is based on the \mathcal{EL} family of DLs; OWL 2 RL (or just RL) is based on Datalog; and OWL 2 QL (or just QL) is based on DL-Lite. Standard reasoning tasks, including classification and fact entailment (checking whether an ontology and a dataset entail a given ground atom), are feasible in polynomial time for all profiles, and many highly scalable reasoners have been developed [22,11,2,4].

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Unfortunately, many ontologies fall outside the OWL 2 profiles, and we are forced to resort to a fully-fledged reasoner if a completeness guarantee is required. Even in such cases, the majority of axioms typically still fall within one of the profiles, and the out-of-profile axioms may have little or no influence on the results of classification or query answering. Effectively detecting cases where the additional expressivity is used in a “harmless” way is, however, challenging, since even a single axiom can have a dramatic effect on reasoning outcomes.

In this paper we investigate techniques for rewriting out-of-profile axioms so as to improve reasoner performance. All rewritings are polynomial and preserve classification and fact entailment reasoning outcomes. In Section 3, we consider rewritings that are applicable to *SHOIQ*—a DL that covers OWL DL and most of OWL 2 [10]—and that can transform non-EL axioms into EL by elimination of inverse roles and universal restrictions. If all non-EL axioms can be rewritten, then we can provide completeness guarantees using only an EL reasoner. Otherwise, the rewritings can still improve the performance of fully-fledged reasoners (e.g., by enabling the use of optimisation techniques that are applicable only in the absence of certain constructs) and/or the effectiveness of modular reasoners that combine profile-specific with OWL 2 reasoners, such as MORE [1].

In Section 4, we focus on Horn ontologies and consider rewritings into OWL 2 RL. The RL profile is tightly connected to Datalog, and hence existential restrictions $\exists R.C$ occurring positively in axioms are disallowed, unless C is a singleton nominal $\{o\}$. We show that when R fulfills certain conditions, such concepts $\exists R.C$ can be rewritten into existential restrictions over nominals as accepted in OWL 2 RL; we call such roles R *reuse-safe*. In the limit case where all roles are reuse-safe, the ontology can be polynomially rewritten into RL; if, additionally, the ontology contains no cardinality constraints, it can also be rewritten into EL. Furthermore, if only some roles are reuse-safe, they can be treated by (hyper-)tableau reasoners in an optimised way, potentially reducing the size of the constructed pre-models and improving reasoning times.

We have implemented our rewriting techniques and evaluated their effect on reasoning times over a large repository of ontologies. Our experiments reveal that our EL-ification techniques can lead to substantial improvements in classification times for both standard and modular reasoners. Furthermore, we show that many ontologies contain only reuse-safe roles and hence can be rewritten into RL; thus, highly scalable RL triple stores can be exploited for large-scale data reasoning.

This paper is accompanied by an online technical report.³

2 Preliminaries

A *signature* consists of disjoint countably infinite sets of *individuals* N_I , *atomic concepts* N_C and *atomic roles* N_R . A *role* is an element of $N_R \cup \{R^- \mid R \in N_R\}$. The function $\text{Inv}(\cdot)$ is defined over the set of roles as follows, where $R \in N_R$: $\text{Inv}(R) = R^-$ and $\text{Inv}(R^-) = R$. An *RBox* \mathcal{R} is a finite set of *RIAs* $R \sqsubseteq R'$ and

³ <http://www.cs.ox.ac.uk/isg/TR/safeshoiq.pdf>

transitivity axioms $\text{Tra}(R)$, with R and R' roles. We denote with $\sqsubseteq_{\mathcal{R}}$ the minimal relation over roles in \mathcal{R} s.t. $R \sqsubseteq_{\mathcal{R}} S$ and $\text{Inv}(R) \sqsubseteq_{\mathcal{R}} \text{Inv}(S)$ hold if $R \sqsubseteq S \in \mathcal{R}$. We define $\sqsubseteq_{\mathcal{R}}^*$ as the reflexive-transitive closure of $\sqsubseteq_{\mathcal{R}}$. A role R is *transitive* in \mathcal{R} if there is a role S such that $S \sqsubseteq_{\mathcal{R}}^* R$, $R \sqsubseteq_{\mathcal{R}}^* S$ and either $\text{Tra}(S) \in \mathcal{R}$ or $\text{Tra}(\text{Inv}(S)) \in \mathcal{R}$. A role R is *simple* in \mathcal{R} if no transitive role S exists s.t. $S \sqsubseteq_{\mathcal{R}}^* R$. The set of *SHOIQ concepts* is the smallest set containing A (atomic concept), \top (top), \perp (bottom), $\{o\}$ (nominal), $\neg C$ (negation), $C \sqcap D$ (conjunction), $C \sqcup D$ (disjunction), $\exists R.C$ (existential restriction), $\forall R.C$ (universal restriction), $\leq nS.C$ (at-most restriction), and $\geq nR.C$ (at-least restriction), for $A \in N_C$, C and D SHOIQ concepts, $o \in N_I$, R a role and S a simple role, and n a nonnegative integer. A *literal concept* is either atomic or the negation of an atomic concept. A *TBox* \mathcal{T} is a finite set of GCIs $C \sqsubseteq D$ with C, D concepts. An *ABox* \mathcal{A} is a finite set of assertions $C(a)$ (concept assertion), $R(a, b)$ (role assertion), $a \approx b$ (equality assertion), and $a \not\approx b$ (inequality assertion), with C a concept, R a role and a, b individuals. A *fact* is either a concept assertion $A(a)$ with A atomic, a role assertion, an equality assertion, or an inequality assertion. A knowledge base is a triple $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$. The semantics is standard [10].

We assume familiarity with standard conventions for naming DLs, and we just provide here a definition of the OWL 2 profiles. A SHOIQ KB is:

- EL if (i) it does not contain inverse roles, negation, disjunction, at-most restrictions and at-least restrictions; and (ii) every universal restriction appears in a GCI of the form $\top \sqsubseteq \forall R.C$.
- RL if each GCI $C \sqsubseteq D$ satisfies (i) C does not contain negation as well as universal, at-least, and at-most restrictions; (ii) D does not contain negation (other than \perp), union, existential restrictions (other than of the form $\exists R.\{o\}$), at-least restrictions, and at-most restrictions with $n > 1$.
- QL if it does not contain transitivity and for each GCI $C \sqsubseteq D$ (i) C is either atomic or $\exists R.\top$; (ii) D is of the form $\prod_{i=1}^n B_i$ with each B_i either a literal concept, or \perp , or of the form $\exists R.A$ with R a role and A either atomic or \top .

Classification of \mathcal{K} is the task of computing all subsumptions $\mathcal{K} \models A \sqsubseteq B$ with $A \in N_C \cup \{\top\}$, and $B \in N_C \cup \{\perp\}$. Fact entailment is to check whether $\mathcal{K} \models \alpha$, for α a fact. Both problems are reducible to knowledge base unsatisfiability.

3 EL-ification of SHOIQ ontologies

In this section, we propose techniques for transforming non-EL axioms into EL. Whenever possible, inverse roles are replaced with fresh symbols and the knowledge base is extended with axioms simulating their possible effects. At the same time, we attempt to transform positive occurrences of universal restrictions into negative occurrences of existential restrictions while inverting the relevant role. Note that our techniques do not rewrite disjunctions and cardinality restrictions; thus, ontologies containing such constructs will not be fully rewritten into EL.

3.1 Preprocessing

Before attempting to rewrite a *SHOIQ* knowledge base \mathcal{K} into EL, we first bring \mathcal{K} into a suitable normal form. Normalisation facilitates further rewriting steps, and it allows us to identify axioms with a direct correspondence in EL. For example, $A \sqcup B \sqsubseteq \neg \forall R. \neg C$ is equivalent to the EL axioms $A \sqsubseteq \exists R.C$ and $B \sqsubseteq \exists R.C$. Furthermore, although $A \sqsubseteq \exists R. \neg B$ is not equivalent to an EL axiom, it can be trivially transformed into the EL axioms $A \sqsubseteq \exists R.X$ and $X \sqcap B \sqsubseteq \perp$ by introducing a fresh symbol X . We therefore introduce a normal form that makes explicit those axioms that are neither logically equivalent to EL axioms, nor can be transformed into EL by means of the trivial introduction of fresh symbols.

Definition 1. *A GCI is normalised if it is of either of the following forms, where each $A_{(i)}$ is atomic or \top , B is atomic, each $C_{(j)}$ is atomic, \perp , or a nominal, R is a role, $n \geq 2$, and $m \geq 1$:*

$$\begin{aligned} \text{(N1)} \quad & \prod_{i=1}^n A_i \sqsubseteq \prod_{j=1}^m C_j; & \text{(N2)} \quad & A \sqsubseteq \exists R.A_i; & \text{(N3)} \quad & \exists R.A \sqsubseteq A_i \\ \text{(N4)} \quad & A \sqsubseteq \geq n R.A_i; & \text{(N5)} \quad & A \sqsubseteq \forall R.B; & \text{(N6)} \quad & A \sqsubseteq \leq m R.A_i \end{aligned}$$

A knowledge base $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ is normalised if \mathcal{A} has only facts and each GCI in \mathcal{T} is normalised. Finally, \mathcal{K} is Horn if $m = 1$ in each axiom **N1** or **N6**.

Note that axioms of type **N2** and **N3**, as well as Horn axioms of type **N1**, are EL. To normalise a knowledge base \mathcal{K} , we proceed in two steps. First, we translate \mathcal{K} into the following disjunctive normal form [15].

Definition 2. *A GCI is in disjunctive normal form (DNF) if it is of the form $\top \sqsubseteq \bigsqcup_{i=1}^n C_i$, where each C_i is of the form B , $\{o\}$, $\exists R.B$, $\forall R.B$, $\geq n R.B$, or $\leq n R.B$, for B a literal concept, R a role, and n a nonnegative integer. A knowledge base $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ is in DNF if all roles in \mathcal{A} are atomic, all concept assertions in \mathcal{A} contain only a literal concept, and each GCI in \mathcal{T} is in DNF.*

DNF normalisation can be seen as a variant of the structural transformation, in which all complex concepts are “flattened” and negations are made explicit (see [15] for details). Once \mathcal{K} is in DNF, we can further normalise by replacing concepts $\neg B$ in restrictions $\forall R. \neg B$, $\exists R. \neg B$, $\geq n R. \neg B$ and $\leq n R. \neg B$ with fresh symbols, bringing the remaining negated concepts to the left in GCIs, and introducing fresh symbols for all restrictions occurring in disjunctions.

Definition 3. *Let \mathcal{K} be a KB. Then, $\Upsilon(\mathcal{K})$ is computed from \mathcal{K} as follows: (i) apply the transformation in [15] to obtain $\mathcal{K}' = (\mathcal{R}', \mathcal{T}', \mathcal{A}')$ in DNF; (ii) replace each assertion $\alpha = \neg A(a)$ in \mathcal{A}' with a fact $X_\alpha(a)$, with X_α fresh, and extend \mathcal{T}' with $X_\alpha \sqcap A \sqsubseteq \perp$; and (iii) apply to \mathcal{T}' the transformation Θ in Figure 1.*

The following proposition establishes the properties of normalisation.

Proposition 1. *Let \mathcal{K} be a KB, then $\Upsilon(\mathcal{K})$ is normalised and can be computed in polynomial time in the size of \mathcal{K} . Furthermore, if \mathcal{K} is EL, then so is $\Upsilon(\mathcal{K})$. Finally, \mathcal{K} is satisfiable iff $\Upsilon(\mathcal{K})$ is satisfiable.*

$$\begin{aligned}
\Theta(\mathcal{T}) &= \bigcup_{\alpha \in \mathcal{T}} \Theta(\alpha) \\
\Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \forall R.B) &= \Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \alpha_B) \cup \{\alpha_B \sqsubseteq \forall R.B\} \\
\Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \forall R.\neg B) &= \Theta(\mathbb{C} \sqcap \alpha_B \sqsubseteq \mathbb{D}) \cup \{\exists R.B \sqsubseteq \alpha_B\} \\
\Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \bowtie nR.B) &= \Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \alpha_B) \cup \{\alpha_B \sqsubseteq \bowtie nR.B\} \\
\Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \geq nR.\neg B) &= \Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \geq nR.\alpha_B) \cup \{\alpha_B \sqcap B \sqsubseteq \top\} \\
\Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \leq nR.\neg B) &= \Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \leq nR.\alpha_B) \cup \{\top \sqsubseteq \alpha_B \sqcup B\} \\
\Theta(\mathbb{C} \sqsubseteq \mathbb{D} \sqcup \neg B) &= \Theta(\mathbb{C} \sqcap B \sqsubseteq \mathbb{D}) \\
\Theta(\alpha) &= \alpha \text{ for any other axiom } \alpha.
\end{aligned}$$

Fig. 1. \mathbb{C} is a conjunction of atomic concepts or \top , \mathbb{D} is a disjunction of concepts C , $\forall R.C$, $\bowtie nR.C$ ($\bowtie \in \{\leq, \geq\}$) or \perp , with C literal, B atomic, and α_B is fresh.

3.2 Rewritable Inverse Roles

Satisfiability of \mathcal{SHOIQ} KBs is NEXPTIME-complete, whereas for \mathcal{SHOQ} it is EXPTIME-complete; thus, in general, inverse roles cannot be faithfully eliminated from \mathcal{SHOIQ} KBs by means of a polynomial transformation. The following example illustrates that an obstacle to rewritability is the interaction between inverses and at-most restrictions.

Example 1. Consider $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$, with $\mathcal{R} = \emptyset$, $\mathcal{A} = \{A(a)\}$, and \mathcal{T} as follows:

$$\mathcal{T} = \{A \sqsubseteq \exists R^-.B; \quad B \sqsubseteq \exists R.C; \quad B \sqsubseteq \leq 1 R.\top\}$$

Note that $\mathcal{K} \models C(a)$. In every model $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, object $a^{\mathcal{I}}$ must be R^- -connected to some $x \in B^{\mathcal{I}}$ (due to the first axiom in \mathcal{T}); also, x must be R -connected to some $y \in C^{\mathcal{I}}$ (due to the second axiom). Then, for the last axiom to be satisfied, $a^{\mathcal{I}}$ and y must be identical; thus, $a^{\mathcal{I}} \in C^{\mathcal{I}}$. Figure 2 a) depicts such a model. Consider now \mathcal{K}' obtained from \mathcal{K} by replacing R^- with a fresh atomic role N_{R^-} . Then, $\mathcal{K}' \not\models C(a)$, and Figure 2 b) depicts a model of \mathcal{K}' not satisfying $C(a)$. Extending \mathcal{K}' with EL axioms to simulate the interaction between inverses and cardinality restrictions (and thus recover the missing entailment) seems infeasible. \diamond

We next propose sufficient conditions for inverse roles to be rewritable in the presence of cardinality constraints. Our conditions ensure existence of a one-to-one correspondence between the *canonical* forest-shaped models of the original and rewritten KBs, and hence disallow cases such as Example 2.⁴

⁴ Roughly speaking, a forest-shaped model of a (normalised) knowledge base is canonical if every fact that holds in the model is “justified” by an axiom or assertion in the knowledge base. In particular, the result of unravelling a pre-model constructed by a (hyper-)tableau algorithm is a canonical forest-shaped model.



Fig. 2. A situation where rewriting away inverse roles leads to missing entailments

Definition 4. Let $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ be a normalised \mathcal{SHOIQ} knowledge base. A (possibly inverse) role R is generating in \mathcal{K} if there exists a role R' occurring in \mathcal{T} in an axiom of type **N2** or **N4** such that $R' \sqsubseteq_{\mathcal{R}}^* R$.

An inverse role S^- is rewritable if for each $X \in \{S, S^-\}$ occurring in an axiom of type **N6** we have that $\text{Inv}(X)$ is not generating in \mathcal{K} .

Intuitively, roles R' in axioms **N2** or **N4** are those “inducing” the edges between individuals and their successors in a canonical model; then, a role R is generating if it is a super-role of one such R' . Our condition ensures that “backwards” edges in a canonical model of \mathcal{K} (i.e., those induced by an inverse role) cannot invalidate an at-most cardinality restriction. In the limit case where all inverse roles in a \mathcal{SHOIQ} KB are rewritable, we can faithfully eliminate inverses and rewrite the KB into \mathcal{SHOQ} by means of a polynomial transformation.

Theorem 1. Let \mathcal{C} be the class of all normalised \mathcal{SHOIQ} ontologies containing only rewritable inverse roles. Then, there exists a polynomial transformation mapping each $\mathcal{K} \in \mathcal{C}$ to an equisatisfiable \mathcal{SHOQ} knowledge base.⁵

Theorem 1 identifies a class of \mathcal{SHOIQ} ontologies for which standard reasoning is feasible in EXPTIME (in contrast to NEXPTIME). This result can also be exploited for optimisation: tableaux reasoners employ pairwise blocking techniques over \mathcal{SHOIQ} ontologies, while they rely on more aggressive single blocking techniques for \mathcal{SHOQ} inputs, which can reduce the size of pre-models.

3.3 The EL-ification Transformation

Before presenting our transformation formally, we provide two motivating examples. First, we show how a rewritable inverse role can be eliminated in the presence of cardinality constraints.

Example 2. Let $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ be the following knowledge base:

$$\begin{aligned} \mathcal{R} &= \{R \sqsubseteq T^-; S \sqsubseteq T^-\} \\ \mathcal{T} &= \{A \sqsubseteq \exists R.B; A \sqsubseteq \exists S.C; A \sqsubseteq \leq 1 T^-. \top; B \sqcap C \sqsubseteq D; \exists R.D \sqsubseteq B\} \\ \mathcal{A} &= \{A(a); T(b, a)\} \end{aligned}$$

⁵ Theorem 1 is given here for presentation purposes: it follows as a corollary of Theorem 3, which we state only after presenting our transformations.

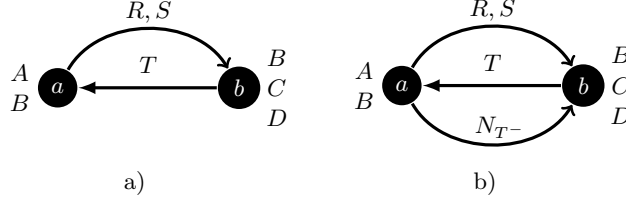


Fig. 3. Rewriting away inverse roles in a KB with cardinality constraints

Figure 3(a) depicts a canonical model for \mathcal{K} . The facts entailed by \mathcal{K} are precisely those that hold in the canonical model. By Definition 4, T^- is rewritable since T is not generating; however, it does not suffice to replace T^- with a fresh N_{T^-} since the resulting KB will no longer entail the facts $R(a, b)$, $S(a, b)$, $B(b)$, $C(b)$, and $D(b)$. Instead, we can extend \mathcal{A} with $T^-(a, b)$, and only then replace T^- with N_{T^-} . The canonical model of the resulting KB is given in Figure 3(b). \diamond

Next, we show how axioms of type **N5**, which involve a universal restriction, can be replaced with EL axioms of type **N3** if the relevant roles are not generating.

Example 3. Consider $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ where $\mathcal{R} = \{R \sqsubseteq S^-\}$, $\mathcal{A} = \{A(a); S(a, b)\}$, and \mathcal{T} is defined as follows:

$$\mathcal{T} = \{A \sqsubseteq \forall S.B; B \sqsubseteq \exists R.C; \exists S.B \sqsubseteq D; C \sqcap D \sqsubseteq \perp\}$$

Clearly, \mathcal{K} is unsatisfiable. Furthermore, it does not contain axioms **N6**, and hence S^- is rewritable. In a first step, we extend \mathcal{K} with logically redundant axioms, which make explicit information that may be lost when replacing inverses with fresh symbols. Thus, we extend \mathcal{T} with $\exists S^-.A \sqsubseteq B$, and $B \sqsubseteq \forall S^-.D$; furthermore, we extend \mathcal{R} with $R^- \sqsubseteq S$; and finally, \mathcal{A} with the assertion $S^-(b, a)$.

An important observation is that S is not generating. As a result, we can dispense with axiom $A \sqsubseteq \forall S.B$. Then we replace S^- with a fresh symbol N_{S^-} and R^- with N_{R^-} . The resulting $\mathcal{K}' = (\mathcal{R}', \mathcal{T}', \mathcal{A}')$ is as follows:

$$\begin{aligned} \mathcal{R}' &= \{R \sqsubseteq N_{S^-}; N_{R^-} \sqsubseteq S\} \\ \mathcal{T}' &= \{\exists N_{S^-}.A \sqsubseteq B; B \sqsubseteq \exists R.C; \exists S.B \sqsubseteq D; B \sqsubseteq \forall N_{S^-}.D; C \sqcap D \sqsubseteq \perp\} \\ \mathcal{A}' &= \{A(a); S(a, b); N_{S^-}(b, a)\} \end{aligned}$$

\mathcal{K}' is unsatisfiable; furthermore it is in EL except for axiom $B \sqsubseteq \forall N_{S^-}.D$. This axiom cannot be dispensed with since S^- is generating, and hence it is needed to propagate information along N_{S^-} -edges in a canonical model. \diamond

We next present our transformation. For simplicity, we first restrict ourselves to $\mathcal{ALCHOIQ}$ KBs; later on, we discuss issues associated with transitivity axioms and show how our techniques extend to \mathcal{SHOIQ} .

Definition 5. Let $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ be a normalised $\mathcal{ALCHOIQ}$ knowledge base. The knowledge base $\Xi(\mathcal{K}) = (\mathcal{R}', \mathcal{T}', \mathcal{A}')$ is obtained as follows:

1. Extension: the knowledge base $\mathcal{K}_e = (\mathcal{R}_e, \mathcal{T}_e, \mathcal{A}_e)$ is defined as follows:
 - \mathcal{R}_e extends \mathcal{R} with an axiom $\text{Inv}(R) \sqsubseteq \text{Inv}(S)$ for each $R \sqsubseteq S$ in \mathcal{R} ;
 - \mathcal{T}_e extends \mathcal{T} with the following axioms:
 - an axiom $\exists \text{Inv}(R).A \sqsubseteq B$ for each axiom $A \sqsubseteq \forall R.B$ in \mathcal{T} where either $\text{Inv}(R)$ is generating, or R is not generating; and
 - an axiom $A \sqsubseteq \forall \text{Inv}(R).B$ for each axiom $\exists R.A \sqsubseteq B$ in \mathcal{T} where $\text{Inv}(R)$ is generating;
 - \mathcal{A}_e extends \mathcal{A} with an assertion $R^-(b, a)$ for each $R(a, b) \in \mathcal{A}$.
2. \mathcal{EL} -ification: $\Xi(\mathcal{K}) = (\mathcal{R}', \mathcal{T}', \mathcal{A}')$ is obtained from \mathcal{K}_e by first removing all axioms $A \sqsubseteq \forall R.B$ in \mathcal{T}_e where R is not generating in \mathcal{T} and then replacing each occurrence of an inverse role that is rewritable in \mathcal{K}_e with a fresh role.

The extension step only adds redundant information, and hence \mathcal{K} and \mathcal{K}_e are equivalent. Making such information explicit is crucial for the subsequent \mathcal{EL} -ification step, where ineffectual axioms involving universal restrictions are removed, and rewritable inverse roles are replaced with fresh atomic roles. The following theorem establishes the properties of the transformation.

Theorem 2. *Let $\mathcal{K}' = \Xi(\mathcal{K})$. The following conditions hold:*

1. \mathcal{K}' is satisfiable iff \mathcal{K} is satisfiable;
2. \mathcal{K}' is of size polynomial in the size of \mathcal{K} ;
3. If \mathcal{K} satisfies all of the following properties, then \mathcal{K}' is \mathcal{EL} :
 - \mathcal{K} is Horn and does not contain axioms of type **N4** or **N6**;
 - each axiom **N5** satisfies either $A = \top$, or R is not generating.
 - each axiom **N3** satisfies either $A = \top$, or $\text{Inv}(R)$ is not generating.

Note that the third condition in the theorem establishes sufficient conditions on \mathcal{K} for the transformed knowledge base \mathcal{K}' to be in \mathcal{EL} . A simple case is when \mathcal{K} is in the \mathcal{QL} profile of OWL 2 , in which case the transformed KB is guaranteed to be in \mathcal{EL} . An interesting consequence of this result is that highly optimised \mathcal{EL} reasoners, such as ELK , can be exploited for classifying \mathcal{QL} ontologies.

Corollary 1. *If \mathcal{K} is a normalised \mathcal{QL} knowledge base, then $\Xi(\mathcal{K})$ is in \mathcal{EL} .*

In many cases our transformation may only succeed in partially rewriting a knowledge base into \mathcal{EL} (c.f. Example 3). Even in these cases, our techniques can have substantial practical benefits (see Evaluation section). As discussed in Section 3.2, in the absence of inverse roles (hyper-)tableau reasoners may exploit more aggressive blocking techniques. Furthermore, modular reasoning systems such as MORe , which are designed to behave better for ontologies with a large \mathcal{EL} subset, are substantially enhanced by our transformations.

3.4 Dealing with Transitivity Axioms

As shown by the following example, the transformation in Definition 5 is not applicable to knowledge bases containing transitivity axioms in the RBox .

Example 4. Consider $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ with $\mathcal{R} = \{R \sqsubseteq R^-; \text{Tra}(R)\}$, $\mathcal{A} = \{A(a)\}$, and $\mathcal{T} = \{A \sqsubseteq \exists R.B; A \sqsubseteq C; \exists R^-.C \sqsubseteq D\}$. Let $\mathcal{K}' = \Xi(\mathcal{K})$, where we assume that the transitivity axiom $\text{Tra}(R)$ stays unmodified in \mathcal{K}' . More precisely, $\mathcal{A}' = \mathcal{A}$, and $\mathcal{R}' = \{R \sqsubseteq N_{R^-}; N_{R^-} \sqsubseteq R; \text{Tra}(R)\}$, and $\mathcal{T}' = \{A \sqsubseteq \exists R.B; A \sqsubseteq C; \exists N_{R^-}.C \sqsubseteq D; C \sqsubseteq \forall R.D\}$. It can be checked that $\mathcal{K} \models D(a)$, but $\mathcal{K}' \not\models D(a)$; thus, a relevant entailment is lost. An attempt to recover this entailment by making N_{R^-} transitive does not solve the problem. \diamond

To address this issue, we eliminate transitivity before applying our transformation in Definition 5. Standard techniques for eliminating transitivity axioms in DLs (e.g., [15]) have the effect of introducing non-Horn axioms. As a result, a Horn knowledge base may not remain Horn after eliminating transitivity. Therefore, we propose a modification of the standard technique that preserves Horn axioms and which is compatible with our transformation in Definition 5.

Definition 6. Let $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ be a normalised *SHOIQ* knowledge base. For each axiom of the form $A \sqsubseteq \forall R.B$ in \mathcal{T} and each transitive sub-role S of R in \mathcal{R} , let $X_{R,B}^S$ be an atomic concept uniquely associated to R, B, S . Furthermore, for each axiom $\exists R.A \sqsubseteq B$ in \mathcal{T} and each transitive sub-role S of R in \mathcal{R} , let $Y_{R,B}^S$ be a fresh atomic concept uniquely associated to R, B, S .

The knowledge base $\Omega(\mathcal{K}) = (\mathcal{R}', \mathcal{T}', \mathcal{A}')$ is defined as follows: (i) \mathcal{R}' is obtained from \mathcal{R} by removing all transitivity axioms; (ii) \mathcal{T}' is obtained from \mathcal{T} by adding axioms $A \sqsubseteq \forall S.X_{R,B}^S$, $X_{R,B}^S \sqsubseteq \forall S.X_{R,B}^S$, and $X_{R,B}^S \sqsubseteq \forall S.B$ for each concept $X_{R,B}^S$, and axioms $\exists S.A \sqsubseteq Y_{R,B}^S$, $\exists S.Y_{R,B}^S \sqsubseteq Y_{R,B}^S$, and $\exists S.Y_{R,B}^S \sqsubseteq B$ for each concept $Y_{R,B}^S$; finally, (iii) $\mathcal{A}' = \mathcal{A}$.

Lemma 1 establishes the properties of transitivity elimination, and Theorem 3 shows that our techniques extend to a *SHOIQ* knowledge base \mathcal{K} by first applying Ω to \mathcal{K} and then Ξ to the resulting KB.

Lemma 1. Let \mathcal{K} be a normalised *SHOIQ* KB. The following holds:

1. $\Omega(\mathcal{K})$ is satisfiable iff \mathcal{K} is .
2. $\Omega(\mathcal{K})$ is a normalised *ALCHOIQ*; furthermore, $\Omega(\mathcal{K})$ is Horn iff \mathcal{K} is Horn.
3. $\Omega(\mathcal{K})$ can be computed in time polynomial in the size of \mathcal{K} .
4. if \mathcal{K} is *EL*, then so is $\Omega(\mathcal{K})$.
5. If an inverse role R^- is rewritable in \mathcal{K} , then it is also rewritable in $\Omega(\mathcal{K})$.

Theorem 3. Let $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ be a normalised *SHOIQ* knowledge base, and let $\mathcal{K}' = \Xi(\Omega(\mathcal{K}))$. Then, \mathcal{K}' satisfies all properties 1 – 3 in Theorem 2.

4 Reuse-safe Roles

We next focus on Horn ontologies, and show how to further optimise reasoning by identifying roles that are “reuse-safe”, and which can thus be treated by (hyper-)tableau reasoners in a more optimised way. Each application of an axiom **N2** or **N4** triggers the generation of fresh individuals in a (hyper-)tableau. If these

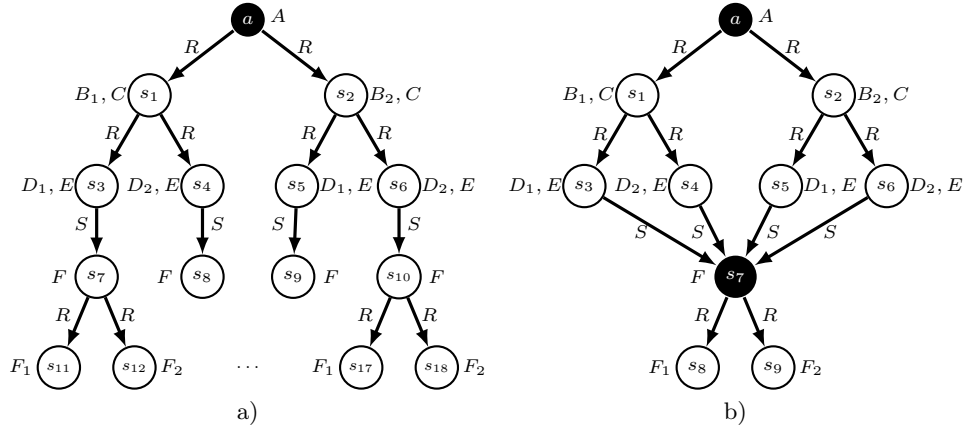


Fig. 4. Decreasing model size by reusing individuals

axioms involve a reuse-safe role, however, we show that reasoners can associate with each such axiom a single fresh nominal, which can be deterministically “reused” whenever the axiom is applied during construction of a pre-model. This may reduce the size of pre-models, and improve reasoning times. Our technique extends the results in [16], which show that for EL ontologies all roles admit reuse, and pre-model size can be bounded polynomially.

Example 5. Consider the following knowledge base $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ where $\mathcal{R} = \emptyset$, $\mathcal{A} = \{A(a)\}$, and \mathcal{T} consists of the following axioms:

$$\begin{aligned} A \sqsubseteq \exists R.B_1 \quad C \sqsubseteq \exists R.D_1 \quad A \sqsubseteq \forall R.C \quad E \sqsubseteq \exists S.F \quad F \sqsubseteq \exists R.F_2 \quad B_1 \sqcap B_2 \sqsubseteq \perp \\ A \sqsubseteq \exists R.B_2 \quad C \sqsubseteq \exists R.D_2 \quad C \sqsubseteq \forall R.E \quad F \sqsubseteq \exists R.F_1 \quad F_1 \sqcap F_2 \sqsubseteq \perp \quad D_1 \sqcap D_2 \sqsubseteq \perp \end{aligned}$$

Since R is generating and \mathcal{K} has no inverses, we have $\Xi(\mathcal{K}) = \mathcal{K}$. Figure 4 a) depicts a canonical model of \mathcal{K} . Role S is reuse-safe since it is not “affected” by non-EL axioms involving universal restrictions. We can exploit this fact to “fold” the model by identifying all nodes with an S -predecessor to a single fresh nominal, as in Figure 4 b). In this way, we can reduce model size. \diamond

Definition 7. Let $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ be a normalised Horn KB. A role R in \mathcal{K} is reuse-safe if either R is not generating or the following conditions hold:

- Each axiom $A \sqsubseteq \leq 1 S.B$ in \mathcal{K} satisfies $R \not\sqsubseteq_{\mathcal{R}}^* S$ and $R \not\sqsubseteq_{\mathcal{R}}^* \text{Inv}(S)$;
- Each axiom $A \sqsubseteq \forall S.B$ in \mathcal{K} with $A \neq \top$ satisfies $R \not\sqsubseteq_{\mathcal{R}}^* S$; and
- Each axiom $\exists S.A \sqsubseteq B$ in \mathcal{K} with $A \neq \top$ satisfies $R \not\sqsubseteq_{\mathcal{R}}^* \text{Inv}(S)$.

If a generating role R is reuse-safe, we can ensure that R -edges in a canonical model of \mathcal{K} are irrelevant to the satisfaction of non-EL axioms in \mathcal{K} . To ensure that (hyper-)tableau algorithms will exploit reuse-safety, and construct succinct “folded” canonical models such as the one in Example 5, we provide the following transformation, which makes the relevant nominals explicit.

Definition 8. Let $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ be a normalised Horn knowledge base.

For each each positive occurrence of a concept $\exists R.B$ (resp. $\geq nR.B$) in \mathcal{K} with R reuse-safe, let $c_{R,B}$ (resp. $c_{i,R,B}$ for $1 \leq i \leq n$) be fresh individual(s). Then, $\Psi(\mathcal{K})$ is KB obtained from \mathcal{K} by:

- replacing each axiom in \mathcal{T} of the form $A \sqsubseteq \exists R.B$, where R is safe, by $A \sqsubseteq \exists R.\{c_{R,B}\}$ and adding the fact $B(c_{R,B})$ to \mathcal{A} , and by
- replacing each axiom of the form $A \sqsubseteq \geq n R.B$, where R is safe, by all $\alpha \in \{A \sqsubseteq \exists R.\{c_{i,R,B}\}, \{c_{j,R,B}\} \sqcap \{c_{k,R,B}\} \sqsubseteq \perp \mid 1 \leq i \leq n \text{ and } 1 \leq j < k \leq n\}$ and adding the facts $B(c_{i,R,B})$, for $1 \leq i \leq n$, to \mathcal{A} .

The following theorem establishes the correctness of our transformation.

Theorem 4. \mathcal{K} is satisfiable iff $\Psi(\mathcal{K})$ is satisfiable.

In practice, system developers can achieve the same goal as our transformation by making their implementations sensitive to reuse-safe roles: to satisfy an axiom involving existential or an at-least restrictions over such role, a system should reuse a suitable distinguished individual instead of generating a fresh one.

We next analyse the case where all roles in a Horn KB $\mathcal{K} = (\mathcal{R}, \mathcal{T}, \mathcal{A})$ are reuse-safe. In this case, we can show that $\Psi(\mathcal{K})$ is in RL. Furthermore, we can identify a new efficiently-recognisable class of DL knowledge bases that contains both EL and RL, and for which both classification and fact entailment are feasible in polynomial time.

Theorem 5. Let \mathcal{C} be the class of Horn knowledge bases \mathcal{K} such that all roles in \mathcal{K} are reuse-safe. Then, the following conditions hold:

1. Checking whether a SHOIQ KB \mathcal{K} is in \mathcal{C} is feasible in polynomial time;
2. Every EL and RL knowledge base is contained in \mathcal{C} ;
3. $\Psi(\mathcal{K})$ is an RL knowledge base for each $\mathcal{K} \in \mathcal{C}$; and
4. Classification and fact entailment in \mathcal{C} are feasible in polynomial time.

Finally, it is worth emphasising that, although the transformations Ψ in Definition 8 and Ξ in Section 3 are very different and serve rather orthogonal purposes, they are connected in the limit case where all roles are reuse-safe and the ontology does not contain cardinality restrictions.

Proposition 2. Let \mathcal{K} be a normalised Horn KB that does not contain axioms **N4** or **N6**. Then, $\Xi(\Omega(\mathcal{K}))$ is EL iff all roles in \mathcal{K} are reuse-safe.

5 Evaluation

We have implemented the transformations described in Sections 3 and 4, and we have performed a range of classification and data reasoning experiments over both realistic ontologies and standard benchmarks.

5.1 Classification Experiments

For our input data, we used the OWL 2 ontologies in the Oxford Ontology Repository,⁶ which contains 793 realistic ontologies, as well as a “hard” version

⁶ <http://www.cs.ox.ac.uk/isg/ontologies/>

Ontology ID	00018	00352	00448	00461	00463	00470	00660	Fly
Original (HermiT)	76.787	18.679	68.545	2.260	t-out	286.89	102.80	840.014
Normalised (HermiT)	30.730	7.235	41.529	11.768	t-out	318.60	123.71	807.167
EL-ified (HermiT)	9.006	7.953	21.395	1.801	651.884	54.40	17.62	17.361
Original (MORe)	42.292	15.095	5.949	2.515	t-out	258.53	99.93	844.639
Normalised (MORe)	10.521	3.195	5.061	11.442	t-out	293.55	85.42	819.640
EL-ified (MORe)	3.0792	2.650	5.019	1.310	694.046	3.48	17.58	17.409

Table 1. Classification times for representative ontologies (in seconds).

of the FlyAnatomy ontology, which is not yet in the repository. Several of the test ontologies contain a small number of axioms exploiting constructs (such as complex RIAs) not available in *SHOIQ*; in these cases we tested filtered versions of the ontologies where such axioms have been removed.

We tested classification times for the latest versions of HermiT (v.1.3.8) and MORe (v.0.1.5) using their standard settings. All experiments were performed on a laptop with 16 GB RAM and Intel Core 2.9 GHz processor running Java v.1.7.0_21, with a timeout set to 3,000s.

EL-ification Experiments. Out of the 793 ontologies in the corpus, we selected those 70 that contain inverse roles, and which HermiT takes at least 1s to classify. For each test ontology \mathcal{K} we have computed a normalised version $\mathcal{Y}(\mathcal{K})$ and an EL-ified version \mathcal{K}' (see Section 3), and have compared classification times for HermiT and MORe on each version.

We found that 50 out of the 70 test ontologies contained only rewritable inverse roles, which could be successfully eliminated using our transformations, and 4 of these ontologies could be fully rewritten into EL. Of these 50 ontologies, 6 could not be classified by HermiT even after EL-ification; however, HermiT succeeded on 2 EL-ified ontologies that could not be classified in their original form. For the remaining 42 ontologies, normalisation alone leads to a slight deterioration in average performance due to the introduction of new class names (which HermiT must classify); however, EL-ification improves HermiT’s performance by an average factor of approximately 3. We believe that this improvement is due to HermiT being able to use single blocking instead of pairwise blocking.

Like HermiT, MORe failed on 8 of the original ontologies, but succeeded on two of these after EL-ification. With the remaining 42, as for HermiT, normalisation alone leads to a slight deterioration in performance, but EL-ification improves performance by an average factor of approximately 6. The larger improvement can be explained by the fact that many axioms are rewritten into EL, and hence MORe can delegate a greater part of the computational work to ELK. Table 1 presents results for some representative cases.

Finally, as already mentioned, our test corpus contains 20 ontologies with non-rewritable inverse roles. As expected, in these cases we obtained no consistent improvement since the presence of inverses forces HermiT to use pairwise blocking; furthermore, in some cases the transformation negatively impacts performance, as it adds a substantial number of axioms to simulate the effect of

inverse roles. Hence, it seems that our techniques are clearly beneficial only when all inverse roles are rewritable.

Reuse Safety. From the 793 ontologies in the corpus, we identified 174 Horn ontologies that do not fall within any of the OWL 2 profiles. We have applied our transformation in Definition 8 to these ontologies and found that 53 do not contain unsafe roles and hence are rewritten into RL. Furthermore, we found that in the remaining ontologies 89% of the roles were reuse-safe, on average. We have tested classification times with HerMiT over the transformed ontologies, but found that the transformation had a negative impact on performance. This is explained by the fact that our transformation introduces nominals. In the presence of nominals, HerMiT disables *anywhere blocking*—a powerful technique that makes nodes blockable by any other node in the tableau (and not just by its ancestors). As mentioned in Section 4, it would be more effective to implement safe reuse as a modification of HerMiT’s calculus; this, however, implies non-trivial modifications to the core of the reasoner, which is left for future work.

5.2 Data Reasoning Experiments

We have used the standard LUBM benchmark, which comes with an ontology about academic departments and a dataset generator parameterised by the number of universities for which data is generated (LUBM(n) denotes the dataset for n universities). The LUBM ontology is not in RL, as it contains axioms of type **N2**; however, all roles in LUBM are reuse-safe and hence we rewrote it into RL using the transformation in Definition 8. For each dataset, we recorded the times needed to compute the instances of all atomic concepts in the ontology. We compared HerMiT over the original ontology and the RL reasoner RDFox⁷ over the transformed ontology. HerMiT took 3.7s for LUBM(1), and timed out for LUBM(5). In contrast RDFox only required 0.2s for LUBM(1), 1.5s for LUBM(10), and 7.4s for LUBM(20). These results suggest the clear benefits of transforming an ontology to RL and exploiting highly scalable reasoners such as RDFox.

6 Related Work

The observation that many ontologies consist of a large EL “backbone” and a relatively small number of non-EL axioms is exploited by the modular reasoner MORE [1] to delegate the bulk of the classification work to EL reasoner ELK [11]. Modular reasoning techniques, however, are sensitive to syntax and all non-EL axioms (as well as those “depending” on them) must be processed by a fully-fledged OWL reasoner. Ren et al. propose a technique for approximating an OWL ontology into EL [17]; this approximation, however, is incomplete for classification and hence valid subsumptions might be lost.

Several techniques for inverse role elimination in DL ontologies have been developed. Ding et al. [7] propose a polynomial reduction from *ALCI* into *ACC*,

⁷ <http://www.cs.ox.ac.uk/isg/tools/RDFox/>

which is then extended in [6] to *SHOI*. Similarly, Song et al. [19] propose a polynomial reduction from *ALCHI* to *ALCH* KBs to optimise classification. In all of these approaches inverse roles are replaced with fresh symbols and new axioms are introduced to compensate for the loss of implicit inferences. These approaches, however, are not applicable to KBs with cardinality restrictions; furthermore, inverse role elimination heavily relies on the introduction of universal restrictions, and hence they are not well-suited for EL-ification. Calvanese et al. [5] propose a transformation from *ALCFI* knowledge bases to *ALC* which is sound and complete for classification; this technique exhaustively introduces universal restrictions to simulate at-most cardinality restrictions and inverse roles, and hence it is also not well-suited for EL-ification; furthermore, this technique is not applicable to knowledge bases with transitive roles or nominals. Finally, Lutz et al. study rewritability of first-order formulas into EL as a decision problem [13]; the rewritings studied in [13], however, require preservation of logical equivalence, whereas ours preserve satisfiability.

The techniques described in Section 4 extend the so-called combined approach to query answering in EL [12,20]. They are also related to are strongly related to individual reuse optimisations [16], where to satisfy existential restrictions a (hyper-)tableau reasoner tries to reuse an individual from the model constructed thus far. Individual reuse, however, may introduce non-determinism in exchange for a smaller model: if the reuse fails (i.e., a contradiction is derived), the reasoner must backtrack and introduce a fresh individual. In contrast, in the case of reuse-safe roles reuse can be done *deterministically* and hence model size is reduced without the need of backtracking.

Finally, Zhou et. al use a very similar transformation as ours to strengthen ontologies and overestimate query answers [23]. It follows from Theorem 5 that the technique in [23] leads to exact answers to atomic queries for Horn ontologies where all roles are reuse-safe.

7 Conclusions and Future Work

In this paper, we have proposed novel techniques for rewriting ontologies into the OWL 2 profiles. Our techniques are easily implementable as preprocessing steps in DL reasoners, and can lead to substantial improvements in reasoning times. Furthermore, we have established sufficient conditions for ontologies to be polynomially rewritable into the EL and RL profiles. Thus, for the class of ontologies satisfying our conditions reasoning becomes feasible in polynomial time. There are many avenues to explore for future work. For example, we will investigate extensions of our EL-ification techniques that are capable of rewriting away disjunctive axioms. Furthermore, we are planning to implement safe reuse in Hermit and evaluate the impact of this optimisation on classification.

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