Consequence based Procedure for Description Logics with Self Restriction

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Abstract. We present a consequence based classification procedure for the description logics with self restriction constructor. Due to the difficulty of constructing a concept inclusion model for self restriction, we use a different proof by showing that all the completion rules can simulate all the corresponding ordered resolution inferences.

1 Introduction

Description logics (DLs) [2] are a family of logic-based formal languages, which provide theoretical foundation for ontology languages, such as OWL 2, the ontology language for the Semantic Web of W3C recommendation[1]. DLs serve as the basis for modeling and reasoning of ontologies. One of the key DL reasoning tasks is ontology classification, whose goal is to compute the hierarchical representation of subclass relations between the concepts in an ontology.

Most of the currently-available ontology reasoners are based on model building procedures such as the tableau [6] and the hyper-tableau [11] calculi. Such procedures classify an input ontology by iterating over all necessary pairs of concepts, and trying to build a model of the ontology that violates the subsumption relation between them. Due to the unnecessary nondeterminism and the construction of large models, tableau methods usually cannot be scalable. Although hyper-tableau method improves the performance significantly, it is too complex to deal with some tractable fragments of DLs efficiently[2]. Instead of building counter models for candidate subsumption relations, the reasoning procedures for tractable DLs, such as $\mathcal{EL}$-family, were discovered to be able to derive subsumption consequences explicitly using inference rules. These rules are designed to produce all implied subsumption relations, while guaranteeing that only a bounded number of axioms is derived. This method is often called as completion rules based procedure or consequence based procedure.

Completion rule based algorithm was firstly introduced for $\mathcal{EL}^{++}$ in [1]. Later on, researchers extended it to Horn-$\mathcal{SHIQ}$ [7] (known as CB[3] reasoner), Horn-$\mathcal{SROIQ}$ [13] and $\mathcal{ALCH}$ [18], which is even beyond Horn DLs. Recently, researchers achieved to perform the consequence based inference in a concurrent

[1] http://www.w3.org/TR/owl2-overview/
[2] See the experiment comparison in [7].
[3] http://code.google.com/p/cb-reasoner/

way [9,8]. The concurrent classification reasoner ELK[4], with its availability of multi-core and multi-processor, shows a substantial speedup by beating all the other currently existing reasoners for classifying SNOMED CT[5] ontologies.

This paper provides a supplemental work for consequence based procedures by extending the availability for self restriction constructor. Since consequence based procedures are closely related to resolution procedure [10,4], it is not difficult to find the completion rules for self restriction. By observing the resolution inference, one can easily establish the completion rules, such as $A \sqsubseteq \exists R.\mathsf{Self}$ and $\exists R.B \sqsubseteq C$ can imply $A \sqcap B \sqsubseteq C$. The reason is that the corresponding resolution inference is by resolving $\neg A(x) \vee R(x,x)$ and $\neg R(x,y) \vee \neg B(y) \vee C(x)$ to produce $\neg A(x) \vee \neg B(x) \vee C(x)$. The relationship between the two can be seen as that consequence based procedure only performs the necessary inferences of ordered resolution procedure. The latter usually produces a large number of irrelevant clauses, which leading to inefficiency in practise.

Traditional proofs [1,7,9] for consequence based procedures hardly work for self restriction, because they are usually based on canonical model construction of concept inclusion. For example, one usually interprets a concept by all its subconcepts and interprets a role by the pair of two concepts[1,7,9], i.e., $A^{\mathcal{I}} = \{C | C \sqsubseteq A\}$ and $R^{\mathcal{I}} = \{\langle A, C \rangle | A \sqsubseteq \exists R.C\}$. Such proofs work well for existential restriction and universal restriction due to their semantics. But inference for self restriction needs unifying variables, because its semantic is based on variables rather than concepts. Therefore, we apply an alternative kind of proof.

The structure of this paper is as follows. Section 2 describes some preliminaries of description logics and resolution procedure. Section 3 presents the completion rules for $\mathcal{ELH}(\mathsf{Self})$. Section 4 extends the algorithm to deal with Horn-$\mathcal{SHI}(\mathsf{Self})$. We will briefly discuss some possible extensions in Section 5. Finally we conclude.

2 Preliminaries

In this section we define the description logic $\mathcal{ELH}(\mathsf{Self})$ and Horn-$\mathcal{SHI}(\mathsf{Self})$, as well as their fragment $\mathcal{ELH}$. Since we only focus on TBox classification task, we will not consider ABox assertions in this paper.

2.1 Description Logics

A signature of $\mathcal{ELH}(\mathsf{Self})$ is a tuple $\Sigma = \langle \mathsf{N}_C, \mathsf{N}_R \rangle$ of mutually disjoint countably infinite sets of *concept names*, *role names*.

The syntax and semantics of $\mathcal{ELH}(\mathsf{Self})$ is summarized in Table 1. The set of $\mathcal{ELH}(\mathsf{Self})$ concepts is recursively defined using the concept constructors given in the upper part of Table 1. The terminology is a set $\mathcal{O}$ of axioms defined in the lower part of Table 1.

[4] http://code.google.com/p/elk-reasoner/
[5] http://www.ihtsdo.org/snomed-ct/

Table 1. Semantics of $\mathcal{ELH}(\mathsf{Self})$

Concept constructor	Syntax	Semantics
top concept	$\top$	$\triangle^{\mathcal{I}}$
bottom concept	$\bot$	$\emptyset$
atomic concept	C	$C^{\mathcal{I}}$
conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
existential restriction	$\exists R.C$	$\{x \in \triangle^{\mathcal{I}} \mid y \in \triangle^{\mathcal{I}} : \langle x, y \rangle \in R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\}$
self restriction	$\exists R.\mathsf{Self}$	$\{x \in \triangle^{\mathcal{I}} \mid \langle x, x \rangle \in R^{\mathcal{I}}\}$
Axioms	**Syntax**	**Semantics**
concept inclusion (GCI)	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
role inclusion	$R \sqsubseteq T$	$R^{\mathcal{I}} \subseteq T^{\mathcal{I}}$
$C, D \in \mathsf{N}_C,\ R, T \in \mathsf{N}_R$		

The semantics of $\mathcal{ELH}(\mathsf{Self})$ is defined using interpretations. An *interpretation* is a pair $\mathcal{I} = (\triangle^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where $\triangle^{\mathcal{I}}$ is a non-empty set called *the domain of the interpretation* and $\cdot^{\mathcal{I}}$ is *the interpretation function*, which assigns to every $A \in \mathsf{N}_C$ a set $A^{\mathcal{I}} \subseteq \triangle^{\mathcal{I}}$, and to every $R \in \mathsf{N}_R$ a relation $R^{\mathcal{I}} \subseteq \triangle^{\mathcal{I}} \times \triangle^{\mathcal{I}}$. An interpretation $\mathcal{I}$ satisfies an axiom α (written $\mathcal{I} \models \alpha$) if the respective condition of the right part in Table 1 holds; $\mathcal{I}$ is a model of an ontology $\mathcal{O}$ (written $\mathcal{I} \models \mathcal{O}$) if $\mathcal{I}$ satisfies every axiom in $\mathcal{O}$. We say that α is a (logical) consequence of $\mathcal{O}$, or is entailed by $\mathcal{O}$ (written $\mathcal{O} \models \alpha$) if every model of $\mathcal{O}$ satisfies α.

$\mathcal{ELH}$ is the fragment of $\mathcal{ELH}(\mathsf{Self})$ by disallowing self restriction. Horn-$\mathcal{SHI}(\mathsf{Self})$ extends $\mathcal{ELH}(\mathsf{Self})$ with role transitivity, inverse role and positive negation, disjunction and universal restriction (left hand of an axiom). However, since positive negation and disjunction can be simulated by conjunction, one can ignore the two constructors in Horn-$\mathcal{SHI}(\mathsf{Self})$. For example, $A \sqsubseteq \neg C$ is equivalent to $A \sqcap C \sqsubseteq \bot$, $A \sqsubseteq B \sqcap C$ is equivalent to two axioms $A \sqsubseteq B$ and $A \sqsubseteq C$.

2.2 Completion Rules for $\mathcal{ELH}$

In [1], a polynomial time classification procedure has been presented for the description logic $\mathcal{EL}^{++}$, which extends $\mathcal{ELH}$ with the nominals, complex role inclusion(role chain) and "safe" concrete domains. The procedure uses a number of completion rules for deriving new concept inclusions. In Table 3, we list the completion rules relevant to $\mathcal{ELH}$[7]. Since the rules are applied to a normalized $\mathcal{ELH}$ ontology $\mathcal{O}$ that is obtained from the input ontology by structural transformation and simplification, we provide the $\mathcal{ELH}$ normal forms in Table 2. In [1], it was shown that the rules IR1-R5 are sound and complete for classication, that is, a concept subsumption $A \sqsubseteq B$ is entailed by $\mathcal{O}$ if and only if it is derivable by these rules.

Table 2. normal forms of $\mathcal{ELH}$ axioms

$A \sqsubseteq \bot$	$\bot \sqsubseteq C$	$A \sqsubseteq C$	$A \sqcap B \sqsubseteq C$	$\exists R.A \sqsubseteq C$	$A \sqsubseteq \exists R.B$	$R \sqsubseteq T$

Table 3. The Completion Rules for $\mathcal{ELH}$

IR1 $\dfrac{}{A \sqsubseteq A}$ IR2 $\dfrac{}{A \sqsubseteq \top}$

CR1 $\dfrac{A \sqsubseteq B \quad B \sqsubseteq C}{A \sqsubseteq C}$

CR2 $\dfrac{A \sqsubseteq B \quad A \sqsubseteq C \quad B \sqcap C \sqsubseteq D}{A \sqsubseteq D}$

CR3 $\dfrac{A \sqsubseteq B \quad B \sqsubseteq \exists R.C}{A \sqsubseteq \exists R.C}$

CR4 $\dfrac{A \sqsubseteq \exists R.B \quad R \sqsubseteq S}{A \sqsubseteq \exists S.B}$

CR5 $\dfrac{A \sqsubseteq \exists R.B \quad B \sqsubseteq C \quad \exists R.C \sqsubseteq D}{A \sqsubseteq D}$

2.3 Ordered Resolution

Ordered resolution [4] is a widely used calculus for theorem proving in first order logic (FOL). The calculus has two parameters, an admissible ordering $\succ$ on literals and a selection function.

An ordering $\succ$ on literals is admissible if (1) it is well-founded, stable under substitutions, and total on ground literals; (2) $\neg A \succ A$ for all ground atoms A; and (3) $B \succ A$ implies $B \succ \neg A$ for all atoms A and B. A literal L is (strictly) maximal with respect to a clause C if there is no other literal $L^{'} \in C$ such that $(L^{'} \succeq L) L^{'} \succ L$. A literal $L \in C$ is (strictly) maximal in C if and only if L is (strictly) maximal with respect to $C \backslash L$. [10]

A *selection function* S assigns to each clause C a subset of negative literals of C (empty possibly); the literals are said to be *selected* if they are in $S(C)$. No other restrictions are imposed on the selection function, i.e., any arbitrary function mapping to negative literals are allowed.

With $\mathcal{R}$ we denote the ordered resolution calculus, where $D \vee \neg B$ is called the main premise. $C \vee A$ is called the side premise, and $C\sigma \vee D\sigma$ is called conclusion:

Ordered Resolution: $$\frac{C \vee A \quad D \vee \neg B}{C\sigma \vee D\sigma}$$

where (1) $\sigma = mgu(A, B)$, (2) $A\sigma$ is strictly maximal with respect to $C\sigma$, and no literal is selected in $C\sigma \vee A\sigma$, (3) $\neg B\sigma$ is either selected in $D\sigma \vee \neg B\sigma$, or it is maximal with respect to $D\sigma$ and no literal is selected in $D\sigma \vee \neg B\sigma$.

For general FOL, there is another rule needed, called *positive factoring*. It resolves two positive literals in one clause. However, since the target DLs in the

Table 4. Translating $\mathcal{ELH}(\mathsf{Self})$ into First Order Logic

Translating Concepts into FOL
$\pi_x(\bot) = \bot$
$\pi_x(\top) = \top$
$\pi_x(C) = C(x)$
$\pi_x(C \sqcap D) = \pi_x(C) \wedge \pi_x(D)$
$\pi_x(\exists R.C) = \exists y.[R(x,y) \wedge \pi_y(C)]$
$\pi_x(\exists R.\mathsf{Self}) = R(x,x)$
Translating Axioms into FOL
$\pi(C \sqsubseteq D) = \forall x : [\pi_x(C) \to \pi_x(D)]$
$\pi(R \sqsubseteq S) = \forall x \forall y : [R(x,y) \to S(x,y)]$
Translating KB into FOL
$\pi(KB) = \bigwedge_{\alpha \in KB} \pi(\alpha)$

paper are both Horn logics, such that the positive factoring rule is not required any more.

Table 4 shows the DL-to-FOL translation for $\mathcal{ELH}(\mathsf{Self})$. The translation is straightforward based on the semantics of DL.

To be noticed, ordered resolution procedure for first order logic is always sound and complete. However, different settings of the parameters can affect the termination of procedure significantly. Therefore, for decidable fragments of FOL, one needs careful tuning of details.

3 $\mathcal{ELH}(\mathsf{Self})$

In this section, we establish the completion rules for the self restriction constructors in $\mathcal{ELH}(\mathsf{Self})$ and prove its soundness and completeness. Instead of the traditional proof by constructing a model of concept inclusion, we show that the completion rules can simulate all the possible ordered resolution inferences.

3.1 Completion Rules for Self Restriction

The following rules Self1 and Self2 are the completion rules for self restriction. Here, we give a informal explanation why they work. The Self1 rule is trivial. For the Self2 rule, recall what we mentioned in Section 1. The first order logic clauses of the two axioms in Self2 are $\neg A(x) \vee R(x,x)$ and $\neg R(x,y) \vee \neg B(y) \vee C(x)$. Via resolving the two clauses by the ordered resolution, $\neg A(x) \vee \neg B(x) \vee C(x)$ can be produced, which is factually $A \sqcap B \sqsubseteq C$. In subsection 3.3, we will show why these rules are sound and complete. For completion rules, since there is only a polynomial number of the axioms in $\mathcal{ELH}(\mathsf{Self})$ KB and all of them can be computed in polynomial time, so we should also show that the ordered resolution procedure for $\mathcal{ELH}(\mathsf{Self})$is in polynomial time.

Table 5. The Completion Rules for Self Restriction in $\mathcal{ELH}(\mathsf{Self})$

$$\mathsf{Self1}\quad \frac{A \sqsubseteq \exists R.\mathsf{Self} \qquad R \sqsubseteq S}{A \sqsubseteq \exists S.\mathsf{Self}}$$

$$\mathsf{Self2}\quad \frac{A \sqsubseteq \exists R.\mathsf{Self} \qquad \exists R.B \sqsubseteq C}{A \sqcap B \sqsubseteq C}$$

Table 6. $\mathcal{ELH}(\mathsf{Self})$-clause types

(1) $\neg A(x)$	(7) $\neg A(x) \vee B(f(x))$
(2) $C(x)$	(8) $\neg A(x) \vee R(x,x)$
(3) $\neg A(x) \vee C(x)$	(9) $\neg R(x,x) \vee A(x)$
(4) $\neg A(x) \vee \neg B(x) \vee C(x)$	(10) $\neg R(x,y) \vee S(x,y)$
(5) $\neg R(x,y) \vee \neg A(y) \vee C(x)$	(11) $\neg A(x) \vee \neg B(f(x)) \vee C(f(x))$
(6) $\neg A(x) \vee R(x, f(x))$	(12) $\neg A(x) \vee \neg B(f(x)) \vee C(x)$

3.2 Resolution Procedure for $\mathcal{ELH}(\mathsf{Self})$

Since our intuitive idea is to simulate all the possible ordered resolution inferences by the completion rules, we first need to show the ordered resolution procedure for $\mathcal{ELH}(\mathsf{Self})$. We first give the definition of the resolution procedure by setting the two parameters, i.e., the predicate order and selection function.

Definition 1. *Let $\mathcal{R}_{DL}$ denote the ordered resolution calculus $\mathcal{R}$ as follows:*

- The literal ordering is an admissible ordering $\succ$ such that $f \succ R \succ A$, for all function symbol f by skolemization, binary predicate symbol P and unary predicate symbol A.
- The selection function selects every negative maximal binary literal in each clause.

The clauses in Table 6 are all the possible clauses occurring during the ordered resolution procedure. We enumerate all possible $\mathcal{R}_{DL}$ inferences between clauses and show that every conclusion is one of clause types of Table 6. With [n, m] $\rightsquigarrow$ [k] we denote an inference between clause type n and m resulting in clause type k. We denote the set of saturated clauses as $\Xi(KB)$.

Lemma 1. *Each $\mathcal{R}_{DL}$ inference, when applied to $\mathcal{ELH}(\mathsf{Self})$-clauses, produces a $\mathcal{ELH}(\mathsf{Self})$-clause type in Table 6. The maximum length of each clause is 3. And the number of clauses different up to variable renaming is polynomial in $|KB|$.*

Proof. The ordered resolution inferences are possible between the following clauses. $[2,3] \rightsquigarrow [2]$, $[2,4] \rightsquigarrow [3]$. $[6,5] \rightsquigarrow [12]$, $[6,10] \rightsquigarrow [6]$. $[7,1] \rightsquigarrow [2]$, $[7,3] \rightsquigarrow [7]$, $[7,4] \rightsquigarrow [11]$. $[8,5] \rightsquigarrow [4]$, $[8,9] \rightsquigarrow [3]$, $[8,10] \rightsquigarrow [8]$.

(11) $\neg A(x) \vee \neg B(f(x)) \vee C(f(x))$ can only resolve with clause $\neg A(x) \vee B(f(x))$ or $B(x)$, and produce clause $\neg A(x) \vee C(f(x))$. Since ordered resolution only resolves on maximal literals, thus literal $\neg A(x)$ in clause type (7) can never participate. In addition, due to that every function symbol is unique after skolemization, there is no other clauses of clause type (7) containing $B(f(x))$. Since $\neg B(f(x))$ in (11) has to resolve with $B(f(x))$ or $B(x)$, then (11) can only resolve with clause $\neg A(x) \vee B(f(x))$ or $B(x)$. For the same reason, (12) $\neg A(x) \vee \neg B(f(x)) \vee C(x)$ can only resolve with clause $\neg A(x) \vee B(f(x))$ or $B(x)$, and produce clause $\neg A(x) \vee C(x)$.

Any other inferences are not applicable. Therefore, every clause is one of the clause types of Table 6, and the maximum length of clauses is 3. Let c be the number of unary predicates, r the number of binary predicates, and f the number of unary function symbols in the signature of $\Xi(KB)$. Then, trivially c, r and f are linear in $|KB|$. Consider now the maximal $\mathcal{ELH}(\mathsf{Self})$-clause of type 5 in Table 6. There are possibly at most rc^2 clauses of type 5, which the number is polynomial in $|KB|$. For other $\mathcal{ELH}(\mathsf{Self})$-clause types, the bounds on the length and on the number of clauses can be derived in an analogous way. Therefore, the number of $\mathcal{ELH}(\mathsf{Self})$-clauses different up to variable renaming is polynomial in $|KB|$.

Corollary 1. *For a $\mathcal{ELH}(\mathsf{Self})$ knowledge base KB, saturating $\Xi(KB)$ by $\mathcal{R}_{DL}$ decides satisfiability of KB and runs in time polynomial in $|KB|$.*

3.3 Soundness and Completeness

Now, we are ready to show the soundness and completeness of $\mathcal{ELH}(\mathsf{Self})$ completion rules.

Lemma 2. *Each $\mathcal{R}_{DL}$ inference by the ordered resolution procedure for $\mathcal{ELH}(\mathsf{Self})$ can be simulated by the corresponding completion rules.*

Proof. $[2,3] \rightsquigarrow [2]$ and $[2,4] \rightsquigarrow [3]$ can be simulated by CR1 and CR2. $[6,10] \rightsquigarrow [6]$ can be simulated by CR4. $[7,1] \rightsquigarrow [2]$ and $[7,3] \rightsquigarrow [7]$ can be simulated by CR5. $[8,5] \rightsquigarrow [4]$, $[8,9] \rightsquigarrow [3]$ and $[8,10] \rightsquigarrow [8]$ can be simulated by Self1, CR1 and Self2 respectively. For $[6,5] \rightsquigarrow [12]$, since we argued that (12) $\neg A(x) \vee \neg B(f(x)) \vee C(x)$ can only resolve with clause $\neg A(x) \vee B(f(x))$ or $B(x)$, and produce clause $\neg A(x) \vee C(x)$, so the $\mathcal{ELH}(\mathsf{Self})$ knowledge base must contain the following axioms, $A \sqsubseteq \exists R.B$, $\exists R.D \sqsubseteq C$ and $B \sqsubseteq D$. Therefore, such inference can be captured by CR5. Similarly, for $[7,4] \rightsquigarrow [11]$, the knowledge base must contain $A \sqsubseteq \exists R.B$, $B \sqcap C \sqsubseteq D$ and $B \sqsubseteq C$, which can be captured by CR5 as well. So, all of the ordered resolution inferences for $\mathcal{ELH}(\mathsf{Self})$ can be simulated by the corresponding completion rules.

Corollary 2. *The completion rules for $\mathcal{ELH}(\mathsf{Self})$ are sound and complete.*

Proof. By lemma 2, we know each ordered resolution inference can be captured by the corresponding completion rules. Since ordered resolution is sound and complete for FOL, hence for DLs. Therefore, the completion rules for $\mathcal{ELH}(\mathsf{Self})$ are sound and complete.

Table 7. Rules for Horn-$\mathcal{SHI}$(Self)

UR1	$\dfrac{A \sqsubseteq \exists R.B \qquad A \sqsubseteq \forall S.C \qquad R \sqsubseteq S}{A \sqsubseteq \exists R.C}$
UR2	$\dfrac{A \sqsubseteq \exists R.B \qquad B \sqsubseteq \forall S.C \qquad R \sqsubseteq S^-}{A \sqsubseteq C}$
UR3	$\dfrac{A \sqsubseteq \exists R.\mathsf{Self} \qquad B \sqsubseteq \forall S.C}{A \sqcap B \sqsubseteq C}$

4 Horn-$\mathcal{SHI}$(Self)

Horn-$\mathcal{SHI}$(Self) extends $\mathcal{ELH}$(Self) by allowing role transitivity, inverse role and positive occurrences of universal restrictions, i.e., Tra(R), $R \sqsubseteq T^-$ and $A \sqsubseteq \forall R.B$. $\mathcal{EL}^{++}$ allows complex role inclusion (role chain), but disallows inverse role and universal restriction constructors for complexity reasons. In [1], it was shown that adding any of the latter two constructors results in a complexity increase from PTime to EXPTime. An intuitive explanation of the exponential blow-up is that resolving axioms of the form $A \sqsubseteq \exists R.B$ and $C \sqsubseteq \forall R.D$ can possibly produce axioms $\bigsqcap A_i \sqsubseteq \exists R.(\bigsqcap Bj)$, where $\bigsqcap A_i$ and $\bigsqcap Bj$ are arbitrary conjunctions of atomic concepts.

In this section, we first introduce the normalization and the well-known technique for transitive role elimination. Then as the structure in previous section, we give out the extra completion rules for Horn-$\mathcal{SHI}$(Self), then describe the ordered resolution procedure and show the soundness and completeness.

4.1 Normalization

The technique in this subsection can be referred in [7]. We denote a concept C is *simple* if it is of the form $\bot$, A, $\exists R.A$, $\exists R.\mathsf{Self}$, $\forall R.A$, where A is an atomic concept. Every Horn-$\mathcal{SHI}$(Self) ontology $\mathcal{O}$ can be transformed into an ontology $\mathcal{O}'$ containing only axioms of the forms $\sqcap A_i \sqsubseteq C$, $R \sqsubseteq T$ and Tra(R).

For each transitive role R, one can eliminate Tra(R) by introducing a triple of axioms for every axioms $\bigsqcap A_i \sqsubseteq \forall R.B$ and every transitive sub-role T of R, i.e., $\bigsqcap A_i \sqsubseteq \forall R.B'$, $B' \sqsubseteq \forall T.B'$ and $B' \sqsubseteq B$, where B' is a fresh atomic concept.

4.2 More Rules

For Horn-$\mathcal{SHI}$(Self), one also needs to add the rules in Table 7. The UR1 and UR2 rules are the relevant rules for inference among existential restriction, universal restriction and inverse role [7]. UR3 is the rule for inference between self restriction and universal restriction.

Table 8. Horn-$\mathcal{SHI}$(Self)-clause types

(1) $\alpha(x) \vee \beta(f(x)) \vee A(x)$	(6) $\neg A(x) \vee R(f(x), x)$
(2) $\alpha(x) \vee \beta(f(x)) \vee A(f(x))$	(7) $\neg A(x) \vee R(x, x)$
(3) $\neg R(x, y) \vee \neg A(y) \vee C(x)$	(8) $\neg R(x, x) \vee A(x)$
(4) $\neg A(x) \vee \neg R(x, y) \vee B(y)$	(9) $\neg R(x, y) \vee S(x, y)$
(5) $\neg A(x) \vee R(x, f(x))$	(10) $\neg R(x, y) \vee S(y, x)$
$\alpha(x)$ is a disjunction $\neg A_1(x) \vee \ldots \vee \neg A_n(x)$ with $A_i \in KB$	
$\beta(x)$ is a disjunction $\neg B_1(x) \vee \ldots \vee \neg B_n(x)$ with $B_i \in KB$	
Disjunctions $\alpha(x)$ and $\beta(x)$ may be empty	
Disjunctions $\alpha(x)$ and $\beta(x)$ may contain same predicates	

4.3 Soundness and Completeness

Still, we first show the resolution procedure for Horn-$\mathcal{SHI}$(Self), but very briefly. We do not need to modify $\mathcal{R}_{DL}$ for Horn-$\mathcal{SHI}$(Self). Table 8 describes all the possible clauses occurring during the procedure.

Lemma 3. *Each $\mathcal{R}_{DL}$ inference, when applied to Horn-$\mathcal{SHI}$(Self)-clauses, produces a Horn-$\mathcal{SHI}$(Self)-clause type in Table 8. The number of different up to variable renaming is exponential in $|KB|$.*

Proof. (sketch) For Horn-$\mathcal{SHI}$(Self), the ordered resolution inferences are possible between the following clauses. $[2, 1] \rightsquigarrow [1]$, if $\beta(f(x))$ is empty, otherwise $[2, 1] \rightsquigarrow [2]$. $[5, 3] \rightsquigarrow [1]$, $[5, 4] \rightsquigarrow [2]$, $[5, 9] \rightsquigarrow [5]$, $[5, 10] \rightsquigarrow [6]$. $[6, 3] \rightsquigarrow [2]$, $[6, 4] \rightsquigarrow [1]$. $[7, 3] \rightsquigarrow [1]$, $[7, 4] \rightsquigarrow [1]$, $[7, 8] \rightsquigarrow [7]$, $[7, 9] \rightsquigarrow [7]$, $[7, 10] \rightsquigarrow [7]$.

Any other inferences are not applicable. Therefore, every clause is one of the clause types of Table 8. The fact of exponential blow-up of the length and number of clauses is trivial by looking at clause type (1). So, it is straightforward to know that saturating Horn-$\mathcal{SHI}$(Self) $\Xi(KB)$ by $\mathcal{R}_{DL}$ decides satisfiability of KB and runs in time exponential in $|KB|$.

Lemma 4. *Each $\mathcal{R}_{DL}$ inference by the ordered resolution procedure for Horn-$\mathcal{SHI}$(Self) can be simulated by the corresponding completion rules.*

Proof. (sketch) $[2, 1] \rightsquigarrow [1]$ and $[2, 1] \rightsquigarrow [2]$ can be simulated by CR1 and CR2. $[5, 3] \rightsquigarrow [1]$ can be simulated by CR5, $[5, 4] \rightsquigarrow [2]$ by UR1, $[5, 9] \rightsquigarrow [5]$ by CR4 and $[5, 10] \rightsquigarrow [6]$ by UR2. $[6, 3] \rightsquigarrow [2]$ and $[6, 4] \rightsquigarrow [1]$ are by CR5 and UR2. The inference with clause type 7 can be simulated by CR4, Self1, Self2 and UR2. So, all of the ordered resolution inferences for Horn-$\mathcal{SHI}$(Self) can be simulated by the corresponding completion rules. Since ordered resolution is a sound and complete procedure for first order logic, hence for Horn-$\mathcal{SHI}$(Self).

Corollary 3. *The completion rules for Horn-$\mathcal{SHI}$(Self) are sound and complete.*

5 Discussion

We have demonstrated the completion rules for the description logics $\mathcal{ELH}(\mathsf{Self})$ and Horn-$\mathcal{SHI}(\mathsf{Self})$. We believe our work can be easily extended to some even more complex DLs. For example, one can extend self restriction for $\mathcal{ALCH}$ [18]. Although $\mathcal{ALCH}$ allows axioms with universal restriction appearing at left hand, axioms containing self restriction can not resolve with these axioms. The reason is that the first order logic clause of left-universal-restriction axioms contain function symbol such that that literal can not unify with self restriction literal. For example, the FOL clause of $\mathcal{ALCH}$ normal form $\forall R.C \sqsubseteq A$ is $\neg R(x, f(x)) \vee \neg C(f(x)) \vee A(x)$. Clause of the axiom with self restriction, such as $\neg A(x) \vee R(x, x)$ cannot resolve with it, because the variable x in $R(x, x)$ and $\neg R(x, f(x))$ cannot unify. In addition, negation and disjunction of concepts, which are allowed in $\mathcal{ALCH}$, do not infer with self restriction. Therefore, we should be able to extend self restriction for $\mathcal{ALCH}$.

We also conjecture that it should be also easily to extend the completion rules for $\mathcal{ELH}(\mathsf{Self})$ to deal with nominals, i.e., $\mathcal{ELHO}(\mathsf{Self})$. Since the DL-to-FOL translation for nominals can introduce equality literal, the calculi for reasoning in equational first-order logic, *paramodulation* or *superposition* [12,10], are also needed. While, since the variables x in self restriction literal $R(x, x)$ always unify with other values together, such that it should not harm the resolution procedure by producing complex clauses. In this sense, one may also extend the algorithm to $\mathcal{ALCOH}(\mathsf{Self})$ and Horn-$\mathcal{SHOI}(\mathsf{Self})$.

When extending with complex RIAs (role chain), the situation becomes much more complicated. To the best of our knowledge, there is no perfect technique to deal with role composition in resolution procedure. Researchers usually force some restriction on the order of role predicate. In [3], the authors applied even more restricted order than the role regularity of OWL 2 $\mathcal{SROIQ}$ [5]. But recently researchers find complex RIAs can be eliminated by formulating as a recursive expansion of universal restrictions [17], which is similar to the encodings of transitivity axioms as we described in section 4.1. Therefore, the completion rules may even extend to $\mathcal{SROIQ}$ by this elimination technique, togethered with the work for $\mathcal{ALCH}$ [18].

6 Conclusions

In this paper, we demonstrate the completion rules for self restriction constructor. We show this in two cases, $\mathcal{ELH}(\mathsf{Self})$ and Horn-$\mathcal{SHI}(\mathsf{Self})$. We believe these rules and proof technique can be extended to the more complex DLs, which will be our future work. We will also explore the completion rules for more DL constructors, like negation, disjunction and conjunction of role [16] and concept product [15].

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