

# An Ontology Design Pattern for Cooking Recipes – Classroom Created

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**Abstract.** We present a description and result of an ontology modeling process taken to the classroom. The application domain considered was cooking recipes. The modeling goal was to bridge heterogeneity across representational choices by developing a content ontology design pattern which is general enough to allow for the integration of information from different web sites. We will discuss the pattern developed, and report on corresponding insights and lessons learned.

## 1 Introduction

The pattern which we will describe in this paper was developed as part of a class which the last named author conducted at Wright State University in Spring 2014. The class was called *Knowledge Representation for the Semantic Web*, and was an entry-level graduate class, with the goal of conveying to the students the fundamentals of Semantic Web knowledge representation languages.

The teacher had been experimenting in the past with a rather “classical” approach to teaching this subject, which essentially followed [1]. In this approach, topics progressed from light-weight to heavy-weight, starting with RDF and RDFS, their semantics and completion algorithms, the OWL syntax and intuitive semantics, followed by the introduction of description logics, their formal semantics and tableaux-based reasoning procedures. Practical modeling exercises progressed from shallow to deep, essentially starting with taxonomies which were then step by step (as material in the class progressed) expanded and adorned with more sophisticated axiomatization. Logical foundations were conveyed without proofs, as in the textbook, with the goal of allowing the students to develop a rough intuition of the logical underpinnings without diving too deeply into the technical details.

While the classes (and the textbook, for that matter) were well received by the students, these classes fell short of one particular goal which the teacher had: most students did *not* develop an intuition for formal semantics and logical deduction, i.e. the understanding of the languages remained at an informal and rather shallow and mostly syntactic level. While this is not necessarily a problem regarding useful learning outcomes for the students, the teacher was motivated to pursue alternative routes.

For the class in Spring 2014, contents and approaches were radically modified. Explicit treatment of RDF and OWL was deferred to a brief overview at

the end of the class, while the bulk of the material taught was on formal logic around description logics and rules (datalog and logic programming), plus some existential rules, with full formal proofs of (un)decidability results and of soundness and completeness of the presented algorithms. The main emphasis was thus to engulf the students in the formal matter, at which point the concrete W3C standards like RDF and OWL became almost afterthoughts, the syntax and semantics of which could be conveyed very quickly in the end as special cases of what had been dealt with on a more foundational level over the course of four months.

A second major change was adopted: The former class project, which was to build an ontology progressively starting from light-weight (taxonomy) to heavy-weight (OWL and rules axioms) was completely dropped. Instead, the whole class met for a 7-hour session (with breaks) to practice collaborative ontology design pattern (ODP) modeling. In the two previous class sessions, the students had received a brief introduction to ODPs, as well as a minimalistic primer on RDF and linked data. The primary examples discussed were the Semantic Trajectory pattern [2], the Cruise pattern [3] and from Linked MDB (linkedmdb.org). The students did know that they would collaboratively model a concrete ODP, but they did not learn before the class session what notion it would be.

For the modeling session, the class was split into two groups, which initially were to work independently. The charge was as follows.

*Design an ontology design pattern which can be used as part of a “recipe discovery” website. The pattern shall be set up such that content from existing recipe websites can “in principle” be mapped to the pattern (i.e., the pattern gets populated with data from the recipe websites). On the discovery website, detailed graph-queries (using the pattern) shall produce recipes from different recipe websites as results.*

Students were instructed to first look at some popular recipe websites, then to formulate, in natural language, some example queries to the website such as “I have cabbage and potatoes, how can I make that into a nice meal in 45 minutes?” They were then to develop a graph structure as basis for a pattern and check that their queries work with these.

In a second stage, the groups were mixed, with one representative from each group changing to the other group. This representative brought the original questions from his original group and both patterns were modified to accommodate the additional types of queries. The groups then added axioms in the form of OWL or rules to constrain the formal semantics. Figures 1 and 2 depict the two patterns the groups came up with, and we will discuss these further in Section 4.

In a third stage, each group was charged with describing how their pattern could be mapped to the other pattern, and asked to report on the problem points, i.e., where and why mapping of parts of the pattern did not really work.

The whole class finally merged all insights into one draft pattern, which is depicted in Figure 3 with only minor modifications – mostly regarding the choice of names for classes and properties, and a more detailed axiomatization. This resulting pattern is laid out in detail in this paper. To the teacher’s own surprise,

the pattern turned out very well, and the class feedbacks were excellent. We will discuss further aspects of the class pattern modeling experience towards the end and discuss the resulting pattern.

## 2 Modeling Recipe

The term *recipe* has several contextual meanings. It can be defined in a general sense as a method to obtain a desired end. When used in the context of cooking, it is generally considered to be a set of instructions on preparing a culinary dish. As such, it could be viewed as an object with properties such as ingredients and time needed. Alternatively, it could be viewed as a *process*, which takes in some input, has a series of steps to be executed, and produces some output. The time taken to execute the steps and the utensils needed also help describe the recipe.

A cooking recipe, apart from the instructions and ingredients, sometimes contain information that help categorize it better for various needs. A recipe could be described as vegetarian or belonging to a particular course and appropriately categorized. Application-specific information may also be needed to tailor the recipe for specific purposes.

Modeling a piece of particular domain knowledge as an ODP typically requires involvement of domain experts. In modeling recipes for this paper, however, the presence of domain experts is not strictly required since information about recipes is widely available online and easily understandable by most lay people. Moreover, we are not so much interested in the actual resulting pattern, but rather on the experience of using ODP as a pedagogical element for teaching Semantic Web knowledge representation languages. In place of sessions with domain experts, a brainstorming session in the class was carried out to gather the information necessary to model recipe in this paper. The resulting pattern was obtained from scratch since the participants were not exposed to previous attempts for modeling cooking recipe or, in general, food and the related notions, some of which we survey below.

Cantais et al. [4] presented a Food ontology, which works together with the Diets and Product ontologies to provide a recommendation to users on a healthy choice of food in accordance to their health condition. They proposed a formalization based on a use case scenario over diabetic patients. Recipe is not modeled explicitly in these ontologies, although the recommender system that uses them can suggest combination of ingredients appropriate to the dieting requirements.

Mota and Agudo [5] presented an ontology of ingredients, which was used as a part of a recipe adaptation system. Recipe is not modeled as a process as in our pattern, but rather, as a simple collection of those ingredients that can be added when needed or removed when unwanted.

Ribeiro et al. [6] described a Cooking ontology as an example how a spoken dialogue system can be enriched with a domain knowledge. The ontology itself is very fine-grained and complex containing more than 1000 classes divided into seven ontology modules. Recipe is modeled here as a combination of preparation, cooking, and presentation phases, each of which consists of a sequence of tasks.

Other relevant notions such as ingredient, utensil, measure, and unit are also included to model various situations, which may involve variation of measures and units, classification of food items (e.g., alcoholic and non-alcoholic), and differences in courses of the food item due to geographical locations. Recipe can thus be seen as a process like in our pattern, although in our case, the pattern is not as fine-grained as this one to retain a relatively high degree of extensibility.

To populate our pattern (or suitable extension thereof) with data, one can in principle use an approach using indentation and tagging structures in XML documents to learn to extract recipe information in the form of semantic annotations from web pages [7].

### 3 Competency Questions

The two groups were first asked to look at existing recipe websites and formulate the kind of questions the information in the websites helped answer. Group A made the assumption that recipe websites would contain information in standard data types and tried to drill down information to the most primitive level. Accordingly, their questions were also more specific. In the following section, we will look at some of the questions each group came up with, and how the two groups modeled similar information differently. Some of the questions that Group A came up with were as follows:

**Question 1** “Breakfast dishes I can prepare with 2 potatoes and 100 grams of wheat flour.”

Here, group A assumed that quantities such as 2 potatoes and 100 grams of wheat flour should be clearly and unambiguously presented in each website. Group A modeled this information using the `FoodItem` and `quantityOfFood` classes. Group B had included ingredient information in their `Ingredient` and `Quantity` classes.

**Question 2** “Gluten-free desserts with less than 100 calories.”

Group A modeled all food categorization in a single class called `FoodClass`. Food classifications such as vegetarian, gluten-free; flavor information such as spicy, sweet; foods with classifications such as Easter eggs, cuisine and course information are modeled by this class. The above question would be answered by Group A’s model using the `FoodClass` and `NutritionInfo` classes, and may be answered by the `Keyword` class in Group B’s model.

**Question 3** “Mexican dishes which do not use chili.”

An interesting aspect of the model of Group A is that both ingredients and food products are modeled as foods with quantities. This will be discussed later in detail. The above question would be answered by the `FoodClass` and `isOptional` property of the `Ingredient` class in Group A’s model. Group B modeled this information using their `Cuisine` class. It might have been more difficult to query

for the absence of an ingredient using Group B's model.<sup>1</sup> Group A was certain that there should be a difficulty level associated with each recipe and included that in their model. They also associated an Author pattern with each of their recipes. This would help answer questions such as the following:

**Question 4** "Easy Gordon Ramsey breakfast dishes."

It was noted that the problem of uniformity in representing levels of difficulty across websites might show up when modeling this concept. Whereas Group A had an Author associated with each recipe, Group B did not. Therefore the above question couldn't be answered by their model. Cooking utensils needed and time to cook were some information that was agreed to be vital to answer questions such as the following:

**Question 5** "Grilled meat in less than 1 hour."

The above utensil and time information were modeled by both groups. Group A used the utensil and Time classes for this and Group B used CookingTool and OWL:Time classes for this. Group B also had some similar ideas as to what ought to be included in a model for recipe websites. They identified flavor, texture, cuisine and serving temperature as essential attributes of any recipe, to answer questions such as the following:

**Question 6** "Spicy Korean beef dishes."

For the above question, Group B's Cuisine, Flavor and Ingredient classes were modeled. Group A would have answered this question with the FoodClass and Ingredient classes.

**Question 7** "Crunchy brownie recipes."

Group B modeled the Texture class and Keyword class to answer queries such as the above, whereas Group A modeled this information using the FoodClass and Keyword classes.

**Question 8** "Cold appetizers."

Group B had a specific ServingTemperature class to answer queries on the temperature the dish should be served at, while Group A modeled such information only with FoodClass and Keyword classes. They also included cooking utensil and difficulty level:

**Question 9** "Baked/Mashed potatoes."

The above question would need information on the utensil to be used to prepare the recipe, since baking involves an oven and mashing presumably requires a masher or some similar utensil, and this information is modeled using the Utensil class by Group A and CookingTool class by Group B.

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<sup>1</sup> (Local) closed world reasoning is of course also required for this type of query.

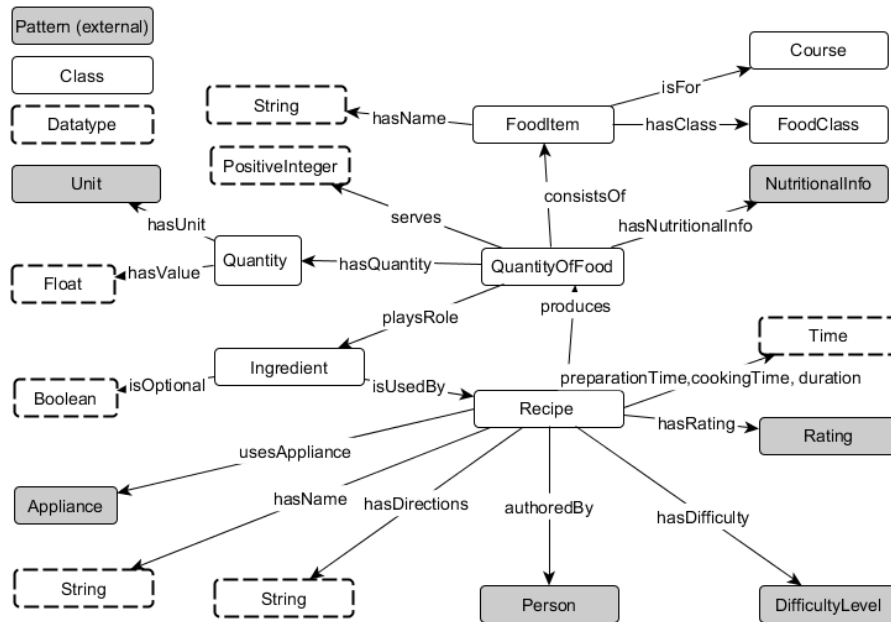


Fig. 1: Recipe modeled by group A.

**Question 10** “Easy desserts with less than 100 calories.”

Both the groups had modeled a DifficultyLevel class and NutritionalInfo class. The above query would be answered by these classes. Some information such as nutritional information, time to cook, utensils needed, difficulty level were identified and modeled similarly by both groups.

#### 4 In-class Modeling – A Discussion

Based on these competency questions, the two groups of students came up with two rather different drafts of a recipe pattern. These drafts are depicted in Figures 1 (Group A) and 2 (Group B).

Initially, both the groups modeled recipe as a class with relationships to other classes. So they defined properties that objects of the class would have. There were ingredients which became a property of the class and there were instructions that was also a property of the class, as was the dish that was produced. With this approach instructions were just another property of the recipe. An alternate way was proposed by the teacher, in which the recipe became a process, and therefore a description of the procedure became now a defining characteristic of it. The students then modified their patterns based on this thinking.

While Group A connected both ingredients and food products as belonging to a single category, which they called FoodItem, Group B continued to view

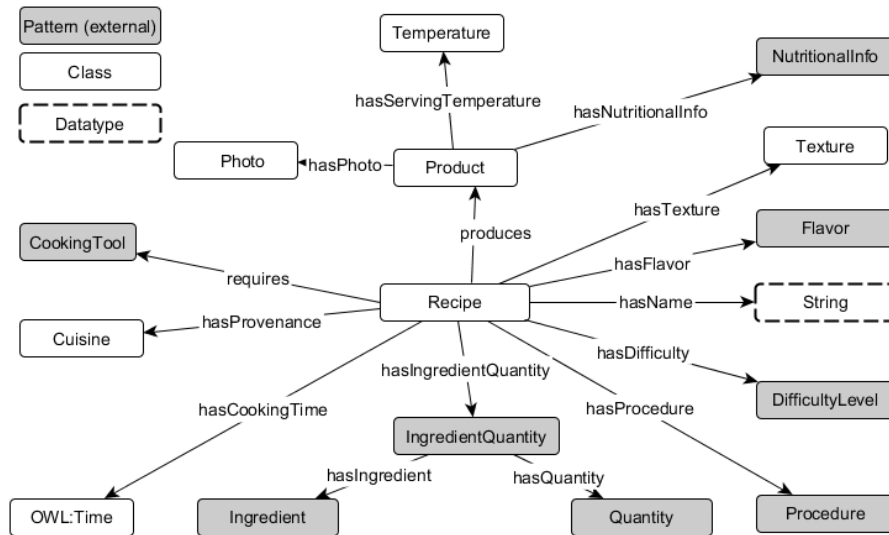


Fig. 2: Recipe modeled by group B.

them separately and modeled them as Product and Ingredient. Both the groups independently came to the conclusion that there had to be some modeling of the concept of quantity to be associated with FoodItems. Group A decided that quantity would in turn have a unit and a value associated with it, whereas Group B did not make that decision. Group A associated all the information related to categorization – based on spice levels, based on cuisine, based on dietary content as all belonging to a single class they called FoodClass. They had a separate class for the course of the dish the recipe was for. Group B decided to maintain separate, the notions of Cuisine, Flavor and Texture. Both the groups independently came up with the idea of NutritionalInfo that had nutritional information about the FoodItem produced as a result of the recipe. Both the groups came up with ideas of time associated with preparation and cooking and the appliances used to make the food item. Both the groups also came up with the idea of a difficulty level associated with the procedure. While the ingredient had an associated property which marked whether it was optional in the design of Group A, Group B had not modeled that information. Also Group A had an author associated with the recipe which Group B did not, while Group B had a Photo associated with the recipe that Group A did not. Group A also had a Rating associated with the recipe that Group B did not.

Of course, the two groups made different modeling choices, and as part of the class session, after producing the initial drafts just discussed, the two groups were asked to attempt to develop mappings, one group from version A to version B, and one group vice versa as the third stage of the class modeling session (see the overview in Section 1). They were charged with developing a loss-less mapping,

if possible, and to report on issues where such a mapping was not possible. This task aligned with the general charge for the modeling session (see Section 1): the charge called for a very general pattern to which content could always be mapped. So difficulties found in the mapping exercise would indicate too specific modeling choices.

A direct mapping could indeed be made between the two models for the following concepts pairs (and corresponding properties): Recipe – Recipe; Ingredient as QuantityOfFood – Ingredient with IngredientQuantity; Quantity – Quantity; Appliance – CookingTool; DifficultyLevel – DifficultyLevel; Name – Name; Directions – Procedure; NutritionalInfo – NutritionalInfo; Time – OWL:Time; FoodClass – Cuisine.

The individual pieces of information about the recipe that each model had which was not in the other model could not be accommodated unless the model was expanded. The author modeled in Group A’s version, the photograph modeled in Group B’s version could not be mapped directly. The unit and value associated with Quantity in Group A’s model would need an expansion of Group B’s model. The Temperature modeled in Group B’s model could not be accommodated in Group A’s model. The Optional Concept associated with Group A’s model could not be directly mapped into Group B’s model. The Rating associated with Group A’s model had no equivalent in Group B’s model. The Course associated with Group A’s model had no equivalent in Group B’s model. The choice of Group A to use a class QuantityOfFood which can act both as recipe ingredient and as recipe product found general agreement, partially because of the perceived conceptual clarity of the modeling, partially because it was realized that sometimes the recipe product becomes in turn an ingredient in another recipe. So the approach by Group A was adopted for the final version.

The mapping attempts furthermore exposed strong ontological commitments regarding some classes used as the range for properties, e.g. demanding a Float as quantity value. In the ensuing discussion it was realized that the quantity value class may be a complex entity, i.e. a pattern in its own right. E.g., recipes may specify a “pinch of salt”, or “salt and pepper to taste”.

Missing information about Photograph in Group A’s model and Author, Rating in Group B’s model was filled by creating a new class called InformationObject which conceptualized information associated with the Recipe. The missing information in Group A’s model about serving temperature, flavor and cuisine and in Group B’s model about course and food class were modeled into another object called FoodClass. By associating a quantity with ingredient, it was decided that the need for an optional indicator field was removed.

The in-class discussion, in particular seeing the different modeling choices, the attempt and failure to produce mappings, and the final consensus to establish the final pattern draft (Figure 3) provided the students with an understanding of the difficulties of making, using, and reusing conceptual models, which would have been much more difficult to convey without a group modeling exercise.



Name	Type	Explanation
<i>hasIngredient</i>	<i>Recipe</i> × <i>QuantityOfFood</i>	An ingredient of the recipe
<i>produces</i>	<i>Recipe</i> × <i>FoodItem</i>	A dish produced by the recipe
<i>consistsOf</i>	<i>QuantityOfFood</i> × <i>FoodItem</i>	The quantity of either an ingredient or dish
<i>classifiedAs</i>	<i>FoodItem</i> × <i>FoodClassification</i>	The classification of an ingredient or dish
<i>hasName</i>	<i>FoodItem</i> × <i>String</i>	The name of an ingredient or dish
<i>hasProcessDescription</i>	<i>Recipe</i> × <i>ProcessDescription</i>	the description of the recipe process
<i>hasTextualDescription</i>	<i>ProcessDescription</i> × <i>Document</i>	The text description of the recipe process
<i>preparationTime, cookingTime</i>	<i>Recipe</i> × <i>TimeInterval</i>	The duration of the recipe process
<i>requires</i>	<i>Recipe</i> × <i>Utensil</i>	A utensil needed for the recipe
<i>hasName</i>	<i>Recipe</i> × <i>String</i>	The name of a recipe
<i>hasInformationObject</i>	<i>Recipe</i> × <i>InformationObject</i>	The information object of a recipe
<i>hasURL</i>	<i>InformationObject</i> × <i>URL</i>	The URL information of a recipe
<i>hasKeyword</i>	<i>InformationObject</i> × <i>String</i>	The keyword information of a recipe
<i>hasAuthor, hasRecommender</i>	<i>InformationObject</i> × <i>Person</i>	The author information of a recipe
<i>hasDifficultyLevel</i>	<i>InformationObject</i> × <i>DifficultyLevel</i>	The difficulty level information of a recipe
<i>hasRating</i>	<i>InformationObject</i> × <i>Rating</i>	The rating of a recipe

Table 1: Basic relations present in the finalized model.

## 5 Finalized Pattern and OWL Formalization

We now present the finalized design pattern, based on the previously described conceptual foundations. The concrete OWL ontology, which was created using Protégé, can be found at <http://www.pascal-hitzler.de/data/RecipeODP.owl>. In order to answer the competency questions, an ontology design pattern needs to distinguish a number of relations. We introduce these abstract relations in Table 1, before formalizing them.

In the following, we respectively discuss the classes and properties within the pattern and formally encode them using the Web Ontology Language (OWL) [8]. We make use of Description Logics (DL) [1] notation, as we believe this improves the readability and understandability of the axioms presented. Note that tractable reasoning is important for producing an efficient implementation of the pattern. A schematic view of the pattern is shown in Figure 3.

**Recipe.** A Recipe is a class which is described as a process. It may take as ingredients instances of QuantityOfFood and also produce instances of QuantityOfFood. The instructions on how to execute the recipe is provided in the ProcessDescription pattern which is assumed to have steps in a time-ordered fashion. A recipe also requires some utensils for execution which is modeled as a Utensil pattern, and requires some amount of time for execution described by a pattern named TimeInterval. These patterns are already assumed to exist in the domain and are left as conceptual elements that are not drilled down in detail for this exercise. Each instance of the recipe class also has a name which is of the String data type.

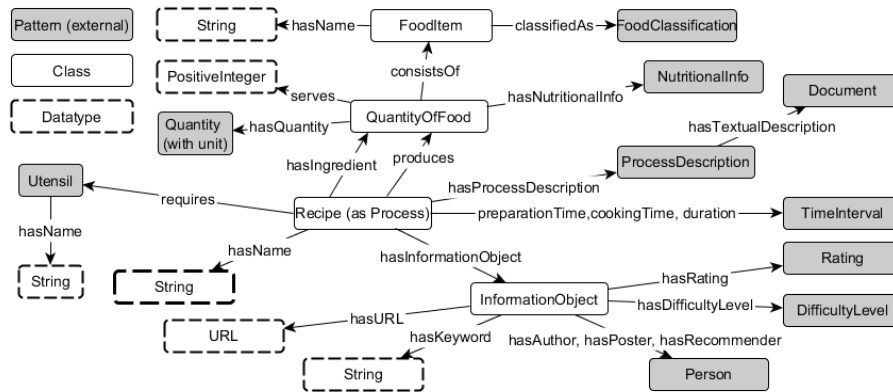


Fig. 3: Recipe as a process modeled along with other patterns (marked grey) needed to describe a recipe.

**QuantityOfFood.** QuantityOfFood is a class that models the quantity of both ingredients and food products produced by the recipe. It can be described as serving a number of people which is of data type positive integer. It also has some nutritional information associated with it, which is modeled as a pattern called NutritionalInfo. The quantity it has is described using a pattern called quantity which is some quantity along with a unit.

**FoodItem.** FoodItem is a class that models an ingredient or food product which in turn has its quantity described by QuantityOfFood. The FoodItem has a name which is modeled using a String data type and has a food classification which in turn is defined as a pattern named FoodClassification. FoodClassification has been modeled in a very broad sense to include such classification as vegetarian, vegan, gluten-free to seasonal, or occasion-specific information such as thanksgiving meals or even meals for a purpose such as lunch-box meals.

**InformationObject.** InformationObject is a class that contains information needed to describe a recipe. Some of its properties are hasURL, hasKeyword (of String data type), hasAuthor, hasPoster, hasRecommender - which is a Person described by an external pattern, and a difficulty level, also modeled as an external pattern. It also includes Rating information, which also could pose challenges, if the rating systems are all not normalized.

**OWL Profile.** Full list of axioms for the pattern are given in Appendix A of the extended technical report available from <http://www.pascal-hitzler.de/pub/2014-recipe-tr.pdf>. From these axioms, we know our recipe pattern falls into the Description Logic  $\mathcal{ELIF}(D)$ . An  $\mathcal{ELIF}$ -Tbox can be reduced to an  $\mathcal{ELI}$ -Tbox whose size is linear in the size of the original one [9].

In  $\mathcal{ELI}$ , subsumption w.r.t. GCIs is ExpTime-complete [10], and  $\mathcal{ELI}$  knowledge bases can be classified by a completion-rule based algorithm<sup>2</sup> [11,9]. However, if we rewrite range restrictions of properties to be of universal scope, e.g.,  $\exists \text{hasIngredient}^- . \top \sqsubseteq \text{QuantityOfFood}$ ,  $\mathcal{ELI}$  can even be reduced to  $\mathcal{EL}$  [12], such that the subsumption checking can be done in polynomial time [10].

## 6 Informal Evaluation

In this section, we illustrate how our pattern can be specialized to the content of specific websites.

For the website *www.allrecipes.com*, most of the information given for recipes can be modeled based on our pattern, except for the addition of a couple of InformationObjects of type image and video. The recipes also have a Review associated with them, but do not contain information on difficulty level.

Another example is the website *www.bettycrocker.com*. The information on this website can be modeled by a slight extension of our pattern to include expert tips to the process description. Also, this website has review information, and no difficulty level.

A third example is taken from *www.epicurious.com*. This website uses information such as the main ingredients, the dietary considerations this recipe would meet – for example, being vegan or high fiber and the season associated with the dish to tag the recipe. Recipes are also categorized. The information in this website could be modeled by of course adding the Review InformationObject, removing the DifficultyLevel and adding in another InformationObject for popularity which lists the number of times the recipe had been downloaded. In all cases, the key page contents can be captured with specializations of our recipe pattern. Further details are provided in Appendix B of the extended technical report available from <http://www.pascal-hitzler.de/pub/2014-recipe-tr.pdf>.

## 7 Conclusions

The class exercise produced a reasonable outcome in terms of the resulting content pattern. The students experienced the power of collaborative modeling to obtain versatile patterns, and overall the experience was resulted in very positive feedbacks.

A lesson learned (for the teacher) is that it seems necessary to convey some amount of thorough logical underpinnings in order to effectively teach introductory ontology modeling: An understanding of logical reasoning with ontology axioms seemed to have helped in avoiding typical beginner’s mistakes regarding ontology modeling, such as confusing part-of relationships with subclass relationships or mistakes arising from vacuous truth via the universal quantifier.

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<sup>2</sup> A practical reasoner for this fragment, called CB reasoner, can be found at <https://code.google.com/p/cb-reasoner/>.

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